

Adaptive Fault-Tolerant Fuzzy Containment Control for Networked Autonomous Surface Vehicles: A Noncooperative Game Approach

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Abstract—This article investigates a containment control problem of networked autonomous surface vehicles (ASVs) with challenges of actuator faults, uncertainties, and environmental disturbances. There exist the individual tasks of networked ASVs and virtual leaders beyond the overall goal. The containment control can be transformed into a noncooperative game problem to balance individual and group objectives. With the noncooperative game theory, the containment task is reformulated as a Nash equilibrium seeking problem of networked ASVs regarded as players. To achieve the Nash equilibrium seeking of ASVs, a noncooperative-game-based fault-tolerant fuzzy containment controller is developed for networked ASVs with an adaptive compensation technique. First, a fuzzy predictor based on the high-order tuner is constructed to tackle the total disturbances. Next, a parameter estimator for unknown faults is designed using the adaptive compensation strategy. With identified disturbances and faults, a desired control law and an intermediate control variable are developed to enable ASVs to operate effectively despite faults. By the Lyapunov functions, theoretical results show that the actions of ASVs can converge to the neighborhood region of the Nash equilibrium. Finally, simulation results are presented to demonstrate the validity of the proposed fault-tolerant containment control method.

Index Terms—Containment control, fuzzy predictor, high-order tuner, networked autonomous surface vehicles (ASVs), noncooperative game.

I. INTRODUCTION

DUE to limited capabilities of a single autonomous surface vehicle (ASV), the distributed cooperative control of multiple ASVs has attracted compelling attention due to its broad maritime applications (e.g., surveillance, exploration, and transportation) and significant advantages (e.g., better efficiency, higher tolerance, and lower network burden) [1], [2], [3]. As a main branch of distributed cooperative control, the objective of containment control is to steer a swarm of plants into a convex hull spanned by multiple leaders and maintain a prescribed geometric pattern [4] and [5]. In terms of containment control, there exist three types of control problems to be presented, namely, trajectory-guided containment control [6], path-guided containment control [7], and target-guided containment control [8].

In the light of aforementioned merits of containment control, there are various research results on the containment control problem of ASVs [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19]. Vehicle uncertainties, environmental disturbances, and modeling errors are inevitable difficulties in implementing the containment control of ASVs. In [9], an echo state network is employed to approximate the nonlinear kinetics through a neural predictor using the measurable velocities. In [10], a neural-state-observer-based containment maneuvering method is developed with only recorded input and output information. In [9] and [10], path maneuvering techniques are incorporated into the containment protocol to decouple the spatial-temporal constraint. In [11], a trajectory-guided output feedback containment control scheme is presented for underactuated vessels via a fuzzy state observer. In [12], the disturbance observer is integrated into the containment controller for ASVs with the prior model. To improve the estimation speed and accuracy, a finite-time disturbance observer is designed in [13]. To achieve the simultaneous estimation for disturbances and states, an extended state observer is used to develop the containment control law [14], [15]. In [14], an artificial potential function is introduced to obtain the collision avoidance and connectivity preservation of containment formation of multiple ASVs. In [15], the safety-critical containment maneuvering method based on the control barrier function is proposed for ASVs with collision-free constraints. The abovementioned continuous-time

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control schemes may result in unnecessary communication consumption when the system tends to be stable [20]. To reduce the network burden, the event-triggered mechanism and the quantized control technique are injected into the distributed control architecture [16], [17], [18], [19]. In [16], a hierarchical sliding-mode-based two-layer coordination architecture under the sampled communication is investigated for multi-ASV containment formation. In [17] and [18], the event-triggered protocol is integrated into the containment controller to degrade the network burden of multivehicle formation. In [19], the quantization strategy of communication level is developed to update the path parameters for fewer data transmissions. It can be seen that the abovementioned results report the containment control methods of multiple ASVs under uncertainties, disturbances, safety constraints, and communication reductions only from a cooperation perspective. However, there exists the noncooperative relationship in the whole group due to energy consumption or competition in a practical scenario.

The game theory has become a research hot spot for first-order and second-order integrators [21], [22], [23], [24], in which Nash equilibrium (NE) seeking strategies are designed to maximize or minimize the objective functions. In recent years, game-based control methods have gradually been extended to marine vehicles [25], [26], [27], [28], [29], [30], [31]. In [26], the static and dynamic formation control methods are investigated for ASVs with partial individual objectives based on the noncooperative game theory. To reduce the communication burden, Fang et al. [25] present a state-quantized location control scheme. In [27], the global optimal formation is further achieved via a regularized game approach. Note that these results [25], [26], [27] are designed using fully known dynamic features and ignoring the impact of winds, waves, and currents. However, modeling errors and environmental disturbances are inevitable for marine vehicles by limiting prior knowledge. To tackle this issue, Wen et al. [28] present a distributed NE formation control method with approximated internal uncertainties from neural networks. Different from these results [25], [26], [27], [28], the backstepping and the dynamic surface control approaches are used to design the NE control law [29], [30], [31]. In [30], a distributed optimal formation of primary–secondary followers is achieved based on the Stackelberg differential graphical game. In [31], the pursuit–evasion game problem of marine vehicles is solved using the minimum and maximum approximation functions. In [30] and [31], the adaptive laws of neural and fuzzy approximations are designed by the gradient descent technique. To improve the convergence rate of adaptive laws, the high-order tuner (HT)-based neural predictor is constructed for the consensus formation of intelligent ASVs [29], in which the acceleration needs to be measured. It is not difficult to find that there are only a few research results of game-based formation control for ASVs, in which the limitations are stated as follows: 1) the control laws developed in [25], [26], [27], [28], [29], [30], and [31] cannot address the containment control with individual and overall objective functions; 2) the fast approximation technique still needs the acceleration information; and 3) vehicle propellers and thrusters need to be well operated.

In a marine environment, propellers and thrusters of ships may suffer from inevitable faults, which will degrade the control performance and even lead to instability unless properly addressed [32], [33]. In [34], a finite-time fault estimator is developed to exactly estimate the lumped disturbances containing the loss-of-effectiveness and additive faults. In [3], a fuzzy state observer is designed to simultaneously estimate the unknown vehicle speed and the fault-involved disturbances. In [35] and [36], an additive fault is regarded as external disturbance to be estimated by the disturbance observer, in which internal uncertainties are identified through neural networks. Note that these fault-tolerant control schemes [3], [32], [33], [34], [35], [36] are designed for the tracking control of a single ASV. It is still open and challenging to seek the NE of the containment control of ASVs with actuator faults and individual objectives.

Motivated by the above discussions, this article develops a noncooperative-game-based fault-tolerant fuzzy containment control method for networked ASVs with uncertainties, environmental disturbances, actuator faults, and individual objectives. The main contributions are stated as follows.

- 1) Compared with containment control methods proposed in [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], and [19] with only considering the cooperation of all individuals, this article develops a containment control method of multiple ASVs under the theoretic framework of noncooperative game. The tradeoff between individual and group objectives can be achieved via the noncooperative-game-based control laws.
- 2) Compared with game-based control methods proposed in [25], [26], [27], [28], [29], [30], and [31] without considering actuator faults, the loss-of-effectiveness and bias faults of ASVs are considered in this noncooperative game. A robust fault-tolerant strategy is incorporated into the noncooperative-game-based containment control law, which can guarantee the NE seeking of ASVs subject to actuator faults.
- 3) In contrast to neural network and fuzzy approximation techniques in [3], [9], [10], [30], [31], [35], [36], [37], [38], [39], [40], [41], [42], [43], and [44] with first-order parameter update laws, we develop an HT-based fuzzy predictor with the potential for accelerated learning. Different from our previous results in [29] and [45], the developed HT-based adaptive law does not require the acceleration information of marine vehicles.

The rest of this article is organized as follows. The preliminaries and problem formulation are introduced in Section II. Section III gives the main results, i.e., the controller development and the stability analysis. Section IV conducts the simulation verification. Finally, Section V concludes this article.

Notations: Throughout this article, $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}^{n \times m}$ denote a real set, an n -dimensional matrix set, and an $(n \times m)$ -dimensional matrix set, respectively. Let $\mathbf{I}_n$ and $\mathbf{0}_n$ be an $n \times n$ identity matrix and a $n \times 1$ zero vector, respectively. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues of a matrix, respectively. A column vector and a block-diagonal matrix are represented as $\text{col}\{\cdot\}$ and $\text{diag}\{\cdot\}$, sequentially. $\|\cdot\|$

and $\|\cdot\|_F$ stand for the Euclidean norm and the Frobenius norm, successively. $\otimes$ is the Kronecker product.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Graph Theory

The network topology of M follower players and $N - M$ leader players is depicted by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. Specifically, $\mathcal{V} = \{\mathcal{V}_F, \mathcal{V}_L\}$ is a vertex set of all players with a follower set $\mathcal{V}_F \triangleq \{1, \dots, M\}$ and a leader set $\mathcal{V}_L \triangleq \{M + 1, \dots, N\}$. Together with $\mathcal{V}$, an edge set is denoted as $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Furthermore, define a neighborhood set of player i as $\mathcal{N}_i = \{\mathcal{N}_{iF}, \mathcal{N}_{iL}\}$ with $\mathcal{N}_{iF} \triangleq \{j \in \mathcal{V}_F | (i, j) \in \mathcal{E}\}$ and $\mathcal{N}_{iL} \triangleq \{k \in \mathcal{V}_L | (i, k) \in \mathcal{E}\}$. The pairs (i, j) and (i, k) stand for successful paths from follower i to follower j and from follower i to leader k , respectively. An adjacency matrix of graph $\mathcal{G}$ is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ with $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$ else $a_{ij} = 0$ ($a_{ii} = 0$). The associated Laplacian matrix is presented as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} \in \mathbb{R}^{N \times N}$ is an in-degree matrix with diagonal elements $\sum_{j \in \mathcal{V}_F} a_{ij} + \sum_{k \in \mathcal{V}_L} a_{ik}$. The graph $\mathcal{G}$ is an undirected graph if $\mathcal{A} = \mathcal{A}^T$.

Assumption 1 (See [46]): The undirected graph is connected for follower and leader players. There exists at least one follower player to catch the information of leader players.

Remark 1: In contrast to follower-leader, behavior-based, and virtual structure methods, the graph-based method is a typical distributed control architecture with promising advantages in decreasing communication burdens, saving computing resources, and improving system robustness [2]. In recent years, graph theory has been widely applied in the formation control of unmanned aerial vehicles [47], [48], ASVs [49], autonomous underwater vehicles [50], and mobile robots [51], to name a few.

B. Fuzzy Logic System (FLS)

The basic configuration of an FLS contains a knowledge base, a fuzzy inference rule, a fuzzifier, and a defuzzifier. Let $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ and y be the input and output of the FLS and define the following “IF-THEN” rules:

RULE^(l): If x_1 is $\mathcal{J}_1^l$, x_2 is $\mathcal{J}_2^l, \dots, x_n$ is $\mathcal{J}_n^l$

THEN: y is $\mathcal{I}^l$, $l = 1, \dots, m$

where m is the number of inference rules. $\mathcal{J}_i^l$ and $\mathcal{I}^l$ represent the fuzzy sets with corresponding membership functions $\mu_{\mathcal{J}_i^l}$ and $\mu_{\mathcal{I}^l}$. Then, it renders that the output

$$y(\mathbf{x}) = \frac{\sum_{l=1}^m \Theta_l \prod_{i=1}^n \mu_{\mathcal{J}_i^l}(x_i)}{\sum_{l=1}^m \prod_{i=1}^n \mu_{\mathcal{J}_i^l}(x_i)} \quad (1)$$

where $\Theta_l = \max_{y \in \mathbb{R}} \mu_{\mathcal{I}^l}(y)$.

Denote $\Theta = \text{col}\{\Theta_l\}$ and $\varphi(\mathbf{x}) = \text{col}\{\varphi_l(\mathbf{x})\}$, $l = 1, \dots, m$ with $\varphi_l(\mathbf{x}) = \prod_{i=1}^n \mu_{\mathcal{J}_i^l}(x_i) / \sum_{l=1}^m \prod_{i=1}^n \mu_{\mathcal{J}_i^l}(x_i)$. Furthermore, the output of the FLS is rewritten as

$$y(\mathbf{x}) = \Theta^T \varphi(\mathbf{x}). \quad (2)$$

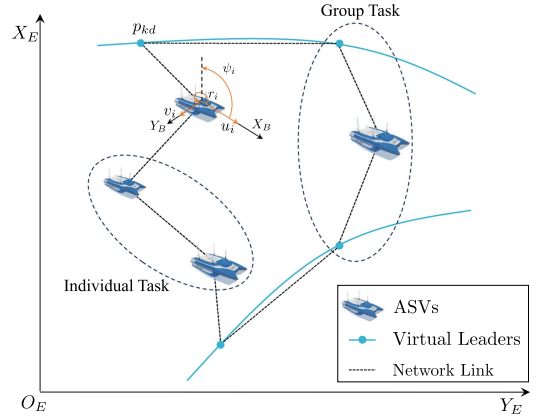


Fig. 1. Illustration for containment pattern of multiple ASVs with individual and group tasks.

TABLE I
DEFINITION FOR THE MATHEMATICAL MODEL OF ASVS

Symbols	Explanations	Units
$\eta_i \in \mathbb{R}^3$	Position and orientation in $\{E\}$	m, m, rad
$\nu_i \in \mathbb{R}^3$	Velocity vector in $\{B\}$	m/s, m/s, rad/s
$M_i \in \mathbb{R}^{3 \times 3}$	Inertial mass matrix	kg, kg, kg·m ²
$C_i \in \mathbb{R}^{3 \times 3}$	Coriolis/centripetal matrix	kg/s, kg/s, kg·m ² /s
$D_i \in \mathbb{R}^{3 \times 3}$	Hydrodynamic damping matrix	kg/s, kg/s, kg·m ² /s
$g_i \in \mathbb{R}^3$	Unmodeled hydrodynamics	N, N, N·m
$\tau_i \in \mathbb{R}^3$	Control torque and moment	N, N, N·m
$\tau_{iw} \in \mathbb{R}^3$	Environmental disturbance	N, N, N·m

Lemma 1 (See [52]): A continuous function $h(\mathbf{x})$ on a compact set Ω can be approximated by the FLS such that $\sup_{\mathbf{x} \in \Omega} |h(\mathbf{x}) - \Theta^T \varphi(\mathbf{x})| \leq \varepsilon^*$ for $\varepsilon^* > 0$.

C. Problem Formulation

This article considers the containment control of M networked ASVs and $N - M$ virtual leaders with individual objectives. Fig. 1 shows the graphical description of the noncooperative game. Herein, the action of ASV i for $i \in \mathcal{N}_F$ is updated by [53]

$$\begin{cases} \dot{\eta}_i = \mathcal{R}(\psi_i) \nu_i \\ M_i \dot{\nu}_i = -C_i(\nu_i) \nu_i - D_i(\nu_i) \nu_i - g_i + \tau_i + \tau_{iw} \end{cases} \quad (3)$$

where the definitions for the mathematical model of ASVs are listed in Table I for clarity. $\mathcal{R}(\psi_i) \in \mathbb{R}^{3 \times 3}$ is a rotation matrix denoted by $\mathcal{R}_i = \mathcal{R}(\psi_i)$ for simplicity, which establishes a map between an earth-fixed reference frame $\{E\}$ and a body-fixed reference frame $\{B\}$. Moreover, $\mathcal{R}_i$ is orthogonal such that

$$\|\mathcal{R}_i\| = 1, \quad \mathcal{R}_i^{-1} = \mathcal{R}_i^T, \quad \dot{\mathcal{R}}_i = \dot{r}_i \mathcal{R}_i S \quad (4)$$

where

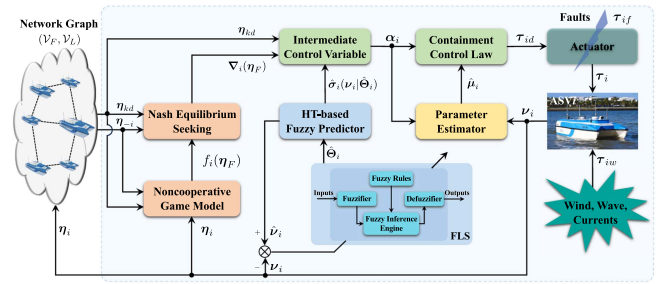
$$\mathcal{R}_i = \begin{bmatrix} \cos \psi_i & -\sin \psi_i & 0 \\ \sin \psi_i & \cos \psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$\tau_{il} = \gamma_{il}\tau_{il,d} + \tau_{il,f}, \quad l = u, v, r \quad (5)$$
$$\begin{aligned} \min_{\boldsymbol{\eta}_i \in \mathbb{R}^3} \quad & f_i(\boldsymbol{\eta}_F) \\ \text{s.t.} \quad & \end{aligned} \quad (3) \quad (6)$$
$$f_i(\eta_i, \eta_{-i}^*) \geq f_i(\eta_i^*, \eta_{-i}^*), \quad i \in \mathcal{V}_F \quad (7)$$
$$\text{Objective 1 : } \min \sum_{j \in \mathcal{N}_{iF}} a_{ij} \|\boldsymbol{\eta}_i - \boldsymbol{\eta}_j\|^2 \quad (8)$$

$$\text{Objective 2 : } \min \sum_{k \in \mathcal{N}_{iL}} a_{ik} \|\boldsymbol{\eta}_i - \boldsymbol{\eta}_{kd}\|^2 \quad (9)$$

Lemma 2 (See [52]): For any $\varkappa > 0$ and $x \in \mathbb{R}$, one has

$$0 \leq |x| - x \tanh\left(\frac{x}{\varkappa}\right) \leq 0.2785\varkappa. \quad (10)$$



Assumption 3: The given trajectory η_{kd} and its derivatives $\dot{\eta}_{kd}$ and $\ddot{\eta}_{kd}$ are bounded such that $\|\eta_{kd}\| \leq \eta_1^* \in \mathbb{R}^+$, $\|\dot{\eta}_{kd}\| \leq \eta_2^* \in \mathbb{R}^+$, and $\|\ddot{\eta}_{kd}\| \leq \eta_3^* \in \mathbb{R}^+$.

In this section, we will develop a noncooperative-game-based fault-tolerant fuzzy containment controller for multiple ASVs subject to actuator faults, unknown internal uncertainties, and external disturbances. The architecture of the proposed method is displayed in Fig. 2.

$$\dot{\nu}_i = \sigma_i(\nu_i, t) + M_i^{-1} \tau_i \quad (11)$$

where $\sigma_i(\cdot) = M_i^{-1}[-C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i - g_i + \tau_{i\omega}]$.

where $\sigma_i(\cdot) = M_i^{-1}[-C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i - g_i + \tau_{i\omega}]$.

For the ASVs with the measurable velocity, a fuzzy predictor is developed to approximate the unknown nonlinear function $\sigma_i(\cdot)$ by

$$\begin{cases} \tilde{\nu}_i = \hat{\nu}_i - \nu_i \\ \dot{\tilde{\nu}}_i = -(K_i^o + 1)\tilde{\nu}_i + \hat{\sigma}_i(\nu_i | \hat{\Theta}_i) + M_i^{-1}\tau_i \end{cases} \quad (12)$$

where $\tilde{\nu}_i$ and $\hat{\nu}_i$ are the estimation error and estimation value of ν_i , respectively. $K_i^o \in \mathbb{R}^{3 \times 3}$ is a diagonal gain matrix with positive elements. $\hat{\sigma}_i(\nu_i | \hat{\Theta}_i) = \hat{\Theta}_i^T \varphi_i(\nu_i)$ denotes the approximation value of $\sigma_i(\nu_i, t)$ with an estimated fuzzy weight matrix $\hat{\Theta}_i \in \mathbb{R}^{3m \times 3}$ and a fuzzy basis function vector $\varphi_i(\nu_i) \in \mathbb{R}^{3m}$.

To obtain the fast convergence, the fuzzy weight $\hat{\Theta}_i$ can be updated by the following HT-based adaptive law [59]:

$$\begin{cases} \dot{\hat{\Theta}}_i = -\Gamma_{i1}\mathcal{T}_i(\hat{\Theta}_i - \Xi_i) \\ \dot{\hat{\Xi}}_i = -\beta_{i1}\varphi_i(\nu_i)\tilde{\nu}_i^T \end{cases} \quad (13)$$

where $\Gamma_{i1} \in \Re$ and $\beta_{i1} \in \Re$ are designed constants. $\mathcal{T}_i = 1 + \phi_i \varphi_i^T(\nu_i) \varphi_i(\nu_i)$, with ϕ_i being a positive parameter satisfying $\phi_i \geq 2\beta_{i1}/\Gamma_{i1}$.

Remark 3: In the existing literature, the adaptive law for weight update is usually designed based on the gradient-based descent strategy, viewed as the first-order tuner. The core idea

behind the HT is the momentum-based updates such that the parameter estimation achieves a faster convergence. See [60] for the detailed analysis.

B. Noncooperative-Game-Based Fault-Tolerant Fuzzy Containment Controller Design

Based on the fuzzy predictor (12) and (13), this section will derive an adaptive fault-tolerant containment control law to balance the noncooperative objectives for a fleet of ASVs.

First of all, the containment task of M ASVs guided by $N - M$ virtual leaders is described as a noncooperative game problem with a cost function

$$f_i(\eta_F) = \frac{1}{2} \sum_{j \in \mathcal{N}_{iF}} a_{ij} \|\eta_i - \eta_j\|^2 + \frac{1}{2} \sum_{k \in \mathcal{N}_{iL}} a_{ik} \|\eta_i - \eta_{kd}\|^2$$

which includes the individual task and swarm task.

To proceed, the partial derivative of $f_i(\eta_F)$ with respect to η_i is taken as follows:

$$\frac{\partial f_i(\eta_F)}{\partial \eta_i} = \sum_{j \in \mathcal{N}_{iF}} a_{ij}(\eta_i - \eta_j) + \sum_{k \in \mathcal{N}_{iL}} a_{ik}(\eta_i - \eta_{kd}).$$

Letting $\nabla(\eta_F) = \text{col}\{\partial f_i(\eta_F)/\partial \eta_i\}$, $i \in \mathcal{V}_F$, it follows that:

$$\nabla(\eta_F) = (\mathcal{L}_1 \otimes \mathbf{I}_3)\eta_F + (\mathcal{L}_2 \otimes \mathbf{I}_3)\eta_L \quad (14)$$

where $\eta_L = \text{col}(\eta_{kd})$, $k \in \mathcal{V}_L$. $\mathcal{L}_1 \in \mathbb{R}^{M \times M}$ and $\mathcal{L}_2 \in \mathbb{R}^{M \times (N-M)}$ are block elements of $\mathcal{L}$ with $\mathcal{L}_1 = \mathcal{D}_1 - \mathcal{A}_1$ and $\mathcal{L}_2 = -\mathcal{A}_2$ satisfying

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 \\ \star & \star \end{bmatrix}, \mathcal{A} = \begin{bmatrix} \mathcal{A}_1 & \mathcal{A}_2 \\ \star & \star \end{bmatrix}$$

$$\mathcal{D} = \begin{bmatrix} \mathcal{D}_1 & \mathbf{0}_{M \times (N-M)} \\ \star & \star \end{bmatrix}$$

where symbol $\star$ implies the irrelevant term in this article. Note that $\mathcal{D}_1$ is a diagonal matrix with positive diagonal elements, and $\mathcal{A}_1$ is a nonnegative symmetric matrix, with 0 being diagonal elements. According to [46], $\mathcal{L}_1$ is a nonsingular M-matrix under Assumption 1. Therefore, there exists a diagonal matrix $\mathbf{P} = \text{diag}\{p_i\} \in \mathbb{R}^{M \times M}$, $i \in \mathcal{V}_F$, with $p_i > 0$ such that $\mathcal{L}_1^T \mathbf{P} + \mathbf{P} \mathcal{L}_1 > 0$.

Let $\eta_F^* \in \mathbb{R}^{3M}$ be the NE solution of cost functions $f_i(\eta_F)$ for $i \in \mathcal{V}_F$ such that $\nabla(\eta_F^*) = \mathbf{0}_{3M}$. Then, the time-varying NE solution from (14) is presented as follows:

$$\eta_F^* = -[\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes \mathbf{I}_3] \eta_L. \quad (15)$$

Notice that the desired control input τ_{id} cannot be directly applied to force ASVs because of the unknown parameter γ_i . Therefore, the adaptive compensation strategy is employed to handle the NE seeking problem of networked ASVs. Now, the desired control input is established as follows:

$$\tau_{id} = \hat{\mu}_i \alpha_i \quad (16)$$

where $\hat{\mu}_i \in \mathbb{R}^{3 \times 3}$ represents the estimation for a matrix $\mu_i = \text{diag}\{\mu_{il}\} \in \mathbb{R}^{3 \times 3}$, $l = u, v, r$ with unknown diagonal constant $\mu_{il} = 1/\gamma_{il}$ satisfying $\mu_i \gamma_i = \gamma_i \mu_i = \mathbf{I}_3$. $\hat{\mu}_i$ is adaptively

updated by a parameter estimator

$$\begin{aligned} \dot{\hat{\mu}}_i = & -\Gamma_{i2} \mathbf{M}_i^{-1} \mathcal{R}_i^T [\mathcal{R}_i \nu_i + \ell_i \dot{\eta}_L + \nabla_i(\eta_F)] \alpha_i^T \\ & - \Gamma_{i2} \beta_{i2} \hat{\mu}_i \end{aligned} \quad (17)$$

where $\Gamma_{i2} \in \mathbb{R}^{3 \times 3}$ is a positive-diagonal matrix. Based on the fuzzy predictor (12), the intermediate control variable $\alpha_i \in \mathbb{R}^3$ is designed as

$$\begin{aligned} \alpha_i = & -\mathbf{M}_i \mathcal{R}_i^T \mathbf{K}_i^c [\mathcal{R}_i \nu_i + \ell_i \dot{\eta}_L + \nabla_i(\eta_F)] \\ & - r_i \mathbf{M}_i \mathbf{S} \nu_i - \mathbf{M}_i \hat{\sigma}_i(\nu_i | \hat{\Theta}_i) - \mathbf{M}_i \mathcal{R}_i^T \phi_i \end{aligned} \quad (18)$$

where $\mathbf{K}_i^c \in \mathbb{R}^{3 \times 3}$ denotes a designed gain matrix. $\nabla_i(\eta_F) = \partial f_i(\eta_F)/\partial \eta_i$. $\ell_i \in \mathbb{R}^{3 \times 3(N-M)}$ is a row vector satisfying $[\ell_1^T, \dots, \ell_M^T]^T = \mathcal{L}_1^{-1} \mathcal{L}_2 \otimes \mathbf{I}_3$. In addition, in (18)

$$\phi_i = \delta \tanh\left(\frac{\delta [\mathcal{R}_i \nu_i + \ell_i \dot{\eta}_L + \nabla_i(\eta_F)]}{\varkappa}\right) \quad (19)$$

where $\delta \in \mathbb{R}^+$ and $\varkappa \in \mathbb{R}^+$ are constants.

C. Stability Analysis

This section will prove that the states of ASVs can converge to the NE solutions for containment tracking.

1) *Predictor Subsystem Analysis:* Let $\tilde{\nu}_F = \text{col}\{\tilde{\nu}_i\}$, $\nu_F = \text{col}\{\nu_i\}$, $\hat{\sigma}_F(\nu_F | \hat{\Theta}_F) = \text{col}\{\hat{\sigma}_i(\nu_i | \hat{\Theta}_i)\}$, $\sigma_F = \text{col}\{\sigma_i\}$, $\varphi_F(\nu_F) = \text{col}\{\varphi_i(\nu_i)\}$, $\mathbf{K}_F^o = \text{diag}\{\mathbf{K}_i^o\}$, $\Gamma_{1F} = \text{diag}\{\Gamma_{i1}\}$, $\mathcal{T}_F = \text{diag}\{\mathcal{T}_i\}$, $\hat{\Theta}_F = \text{diag}\{\hat{\Theta}_i\}$, $\Xi_F = \text{diag}\{\Xi_i\}$, and $\beta_{1F} = \text{diag}\{\beta_{i1}\}$, for $i \in \mathcal{V}_F$. With (12) and (13), it puts into the dynamics of estimation subsystem as

$$\begin{cases} \dot{\tilde{\nu}}_F = -(\mathbf{K}_F^o + \mathbf{I})\tilde{\nu}_F + \hat{\sigma}_F(\nu_F | \hat{\Theta}_F) - \sigma_F \\ \dot{\hat{\Theta}}_F = -\Gamma_{1F} \mathcal{T}_F (\hat{\Theta}_F - \Xi_F) \\ \dot{\Xi}_F = -\beta_{1F} \varphi_F(\nu_F) \tilde{\nu}_F^T. \end{cases} \quad (20)$$

Lemma 3 shows the stability of the subsystem (20).

Lemma 3: The subsystem (20) specified by the dynamics (11), the predictor (12), and the HT-based adaptive law (13) results to be bounded with $\lim_{t \rightarrow \infty} \tilde{\nu}_F = 0$.

Proof: Consider a Lyapunov function candidate as

$$\begin{aligned} V_0 = & \frac{1}{2} \tilde{\nu}_F^T \tilde{\nu}_F + \text{tr}[(\Xi_F - \Theta_F)^T \beta_{1F}^{-1} (\Xi_F - \Theta_F)] \\ & + \text{tr}[(\hat{\Theta}_F - \Xi_F)^T \beta_{1F}^{-1} (\hat{\Theta}_F - \Xi_F)] \end{aligned} \quad (21)$$

and take its derivative along dynamics (20) as follows:

$$\begin{aligned} \dot{V}_0 = & -\tilde{\nu}_F^T [(\mathbf{K}_F^o + \mathbf{I})\tilde{\nu}_F - \hat{\sigma}_F(\nu_F | \hat{\Theta}_F) + \sigma_F] \\ & - \text{tr}[(\Xi_F - \Theta_F)^T \varphi_F(\nu_F) \tilde{\nu}_F^T] \\ & - 2\text{tr}[(\hat{\Theta}_F - \Xi_F)^T \beta_{1F}^{-1} \Gamma_{1F} \mathcal{T}_F (\hat{\Theta}_F - \Xi_F)] \\ & + \text{tr}[(\hat{\Theta}_F - \Xi_F)^T \varphi_F(\nu_F) \tilde{\nu}_F^T]. \end{aligned} \quad (22)$$

Since $\sigma_F = \Theta_F^T \varphi_F(\nu_F) + \varepsilon_F$ yields that $\hat{\sigma}_F(\nu_F | \hat{\Theta}_F) - \sigma_F = \tilde{\Theta}_F^T \varphi_F(\nu_F) - \varepsilon_F$ with $\tilde{\Theta}_F = \hat{\Theta}_F - \Theta_F$ being an estimation error, the derivative of V_0 is deduced as

$$\dot{V}_0 \leq -(\varrho_1 + 1) \|\tilde{\nu}_F\|^2 + \tilde{\nu}_F^T \tilde{\Theta}_F^T \varphi_F(\nu_F) - \tilde{\nu}_F^T \varepsilon_F$$

$$\begin{aligned}
& -\text{tr}[(\Xi_F - \Theta_F)^T \varphi_F(\nu_F) \tilde{\nu}_F^T] \\
& + \text{tr}[(\hat{\Theta}_F - \Xi_F)^T \varphi_F(\nu_F) \tilde{\nu}_F^T] \\
& - 2\text{tr}[(\hat{\Theta}_F - \Xi_F)^T \beta_{1F}^{-1} \Gamma_{1F} (\hat{\Theta}_F - \Xi_F)] \\
& - \text{tr}[(\hat{\Theta}_F - \Xi_F)^T (\hat{\Theta}_F - \Xi_F)] \varphi_F^T(\nu_F) \varphi_F(\nu_F) \quad (23)
\end{aligned}$$

where $\varrho_1 = \lambda_{\min}(\mathbf{K}_F^o)$.

Using the property that $\text{tr}(\mathbf{a}\mathbf{b}^T) = \mathbf{a}^T \mathbf{b}$ for any $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$, one has the following transformation:

$$\tilde{\nu}_F^T \tilde{\Theta}_F^T \varphi_F(\nu_F) = \text{tr} \left[\tilde{\Theta}_F^T \varphi_F(\nu_F) \tilde{\nu}_F \right]. \quad (24)$$

Due to $\tilde{\Theta}_F = (\hat{\Theta}_F - \Xi_F) + (\Xi_F - \Theta_F)$, it follows that:

$$\begin{aligned}
\dot{V}_0 & \leq -(\varrho_1 + 1) \|\tilde{\nu}_F\|^2 - \varrho_2 \|\hat{\Theta}_F - \Xi_F\|_F^2 \\
& + 2\text{tr}[(\hat{\Theta}_F - \Xi_F)^T \varphi_F(\nu_F) \tilde{\nu}_F^T] + \|\tilde{\nu}_F\| \|\varepsilon_F\| \\
& - \|\hat{\Theta}_F - \Xi_F\|_F^2 \|\varphi_F(\nu_F)\|^2 \quad (25)
\end{aligned}$$

where $\varrho_2 = \lambda_{\min}(\beta_{1F}^{-1} \Gamma_{1F})$.

According to $\|\mathbf{A}\mathbf{B}\|_F \leq \|\mathbf{A}\| \|\mathbf{B}\|_F$, $\dot{V}_0$ is put into

$$\begin{aligned}
\dot{V}_0 & \leq -(\varrho_1 + 1) \|\tilde{\nu}_F\|^2 - \varrho_2 \|\hat{\Theta}_F - \Xi_F\|_F^2 \\
& + 2\|\tilde{\nu}_F\| \|\hat{\Theta}_F - \Xi_F\|_F \|\varphi_F(\nu_F)\| + \|\tilde{\nu}_F\| \|\varepsilon_F\| \\
& - \|\hat{\Theta}_F - \Xi_F\|_F^2 \|\varphi_F(\nu_F)\|^2 \quad (26)
\end{aligned}$$

$$\begin{aligned}
& \leq -\varrho_1 \|\tilde{\nu}_F\|^2 - \varrho_2 \|\hat{\Theta}_F - \Xi_F\|_F^2 + \|\tilde{\nu}_F\| \|\varepsilon_F\| \\
& - [\|\tilde{\nu}_F\| - \|\hat{\Theta}_F - \Xi_F\|_F \|\varphi_F(\nu_F)\|]^2. \quad (27)
\end{aligned}$$

Furthermore, one obtains

$$\begin{aligned}
\dot{V}_0 & \leq -\varrho_1 \|\tilde{\nu}_F\|^2 - \varrho_2 \|\hat{\Theta}_F - \Xi_F\|_F^2 + \|\tilde{\nu}_F\| \|\varepsilon_F\| \\
& = -\xi_1 \|\mathbf{E}_1\|^2 + \|\varepsilon_F\| \|\mathbf{E}_1\| \quad (28)
\end{aligned}$$

where $\xi_1 = \min\{\varrho_1, \varrho_2\}$ and $\mathbf{E}_1 = [\|\tilde{\nu}_F\|, \|\hat{\Theta}_F - \Xi_F\|_F]^T$.

From Lemma 1, the approximation error ε_F is bounded and satisfied with $\|\varepsilon_F\| \leq \sqrt{3M}\varepsilon^*$. When $\|\mathbf{E}_1\| \geq \sqrt{3M}\varepsilon^*/c_1\xi_1$ with $0 < c_1 < 1$, it leads to $\dot{V}_0 \leq -\xi_1(1 - c_1)\|\mathbf{E}_1\|^2$.

Similar to [61, Th. 4], it is concluded that the error signals $\tilde{\nu}_F$, $\Xi_F - \Theta_F$, and $\hat{\Theta}_F - \Xi_F$ are uniformly bounded using Barbalat's lemma. Moreover, note that $\|\hat{\Theta}_F\|_F = \|(\hat{\Theta}_F - \Xi_F) + (\Xi_F - \Theta_F)\|_F \leq \|\hat{\Theta}_F - \Xi_F\|_F + \|\Xi_F - \Theta_F\|_F$. Therefore, the error Θ_F is bounded by $\|\hat{\Theta}_F\|_F \leq \theta^* \in \mathbb{R}^+$. $\square$

2) *Control Subsystem Analysis:* In the subsequent analysis, the model (3) is rewritten by (5), (11), and (16) as follows:

$$\begin{cases} \dot{\eta}_F = \mathcal{R}_F \nu_F \\ \dot{\nu}_F = M_F^{-1} \gamma_F \hat{\mu}_F \alpha_F + \hat{\sigma}_F(\nu_F | \hat{\Theta}_F) \\ \quad - \tilde{\Theta}_F^T \varphi_F(\nu_F) + \varepsilon_F + M_F^{-1} \tau_{Ff} \end{cases} \quad (29)$$

where $\alpha_F = \text{col}\{\alpha_i\}$, $\varepsilon_F = \text{col}\{\varepsilon_i\}$, $\tau_{Ff} = \text{col}\{\tau_{if}\}$, $\mathcal{R}_F = \text{diag}\{\mathcal{R}_i\}$, $M_F = \text{diag}\{M_i\}$, $\gamma_F = \text{diag}\{\gamma_i\}$, and $\hat{\mu}_F = \text{diag}\{\hat{\mu}_i\}$ for $i \in \mathcal{V}_F$.

Under Assumptions 1–3, the main result of this article is stated by the following theorem.

Theorem 1: Consider a fleet of networked ASVs with the action dynamics (3), the actuator fault (5), the individual objective (8), and the group objective (9). Under Assumptions 1–3, all error signals of the closed-loop noncooperative-game-based containment system are uniformly ultimately bounded with the fuzzy predictor (12), the adaptive law (13), the fuzzy containment controller (18), and the parameter estimator (17).

Proof: The analysis progress will be completed via the following steps.

Step 1: Construct a Lyapunov function candidate $V_1 = (\eta_F - \eta_F^*)^T (P \otimes I_3) (\eta_F - \eta_F^*)$ and differentiate it along (29) as follows:

$$\begin{aligned}
\dot{V}_1 & = (\eta_F - \eta_F^*)^T (P \otimes I_3) [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L] \\
& + [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L]^T (P \otimes I_3) (\eta_F - \eta_F^*).
\end{aligned}$$

Substituting $\nabla(\eta_F)$ and $\nabla(\eta_F^*)$ into its derivative, one obtains

$$\begin{aligned}
\dot{V}_1 & = (\eta_F - \eta_F^*)^T (P \otimes I_3) [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L \\
& + \nabla(\eta_F)] + [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)]^T \\
& \times (P \otimes I_3) (\eta_F - \eta_F^*) \\
& - (\eta_F - \eta_F^*)^T (P \otimes I_3) [\nabla(\eta_F) - \nabla(\eta_F^*)] \\
& - [\nabla(\eta_F) - \nabla(\eta_F^*)]^T (P \otimes I_3) (\eta_F - \eta_F^*). \quad (30)
\end{aligned}$$

Note that $\nabla(\eta_F) - \nabla(\eta_F^*) = (\mathcal{L}_1 \otimes I_3) (\eta_F - \eta_F^*)$ by (29) and (15). Thus, we have the following inequality:

$$\begin{aligned}
& - (\eta_F - \eta_F^*)^T (P \otimes I_3) [\nabla(\eta_F) - \nabla(\eta_F^*)] \\
& - [\nabla(\eta_F) - \nabla(\eta_F^*)]^T (P \otimes I_3) (\eta_F - \eta_F^*) \\
& = -(\eta_F - \eta_F^*)^T [(P \mathcal{L}_1 + \mathcal{L}_1^T P) \otimes I_3] (\eta_F - \eta_F^*) \\
& - \varrho_3 \|\eta_F - \eta_F^*\|^2 \quad (31)
\end{aligned}$$

with $\varrho_3 = \lambda_{\min}(P \mathcal{L}_1 + \mathcal{L}_1^T P)$.

Then, it puts into

$$\begin{aligned}
\dot{V}_1 & \leq -\varrho_3 \|\eta_F - \eta_F^*\|^2 + 2\lambda_{\max}(P) \|\eta_F - \eta_F^*\| \\
& \times \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\|. \quad (32)
\end{aligned}$$

Step 2: Construct a Lyapunov function candidate V_2 as

$$V_2 = \frac{1}{2} \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\|^2. \quad (33)$$

Taking the derivative of V_2 and substituting (14), (18), and (29), we obtain

$$\begin{aligned}
\dot{V}_2 & = [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)]^T \\
& \{ -K_F^c [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)] \\
& - \mathcal{R}_F [\tilde{\Theta}_F^T \varphi_F(\nu_F) - \varepsilon_F] + \mathcal{R}_F M_F^{-1} \gamma_F \tilde{\mu}_F \alpha_F \\
& - \phi_F + \mathcal{R}_F M_F^{-1} \tau_{Ff} + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \ddot{\eta}_L \\
& + (\mathcal{L}_1 \otimes I_3) \mathcal{R}_F \nu_F + (\mathcal{L}_2 \otimes I_3) \dot{\eta}_L \} \quad (34)
\end{aligned}$$

with $\tilde{\mu}_F = \hat{\mu}_F - \mu_F$, $K_F^c = \text{diag}\{K_i^c\}$, and $\phi_F = \text{diag}\{\phi_i\}$ for $i \in \mathcal{V}_F$.

With the aid of $\nabla(\eta_F^*) = \mathbf{0}_{3M}$ and $\nabla(\eta_F) - \nabla(\eta_F^*) = (\mathcal{L}_1 \otimes I_3)(\eta_F - \eta_F^*)$, V_2 is deduced as

$$\begin{aligned} \dot{V}_2 = & -[\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)]^T [K_F^c \\ & - (\mathcal{L}_1 \otimes I_3)] [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)] \\ & + [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)]^T [-\phi_F \\ & - \mathcal{R}_F \tilde{\Theta}_F^T \varphi_F(\nu_F) + \mathcal{R}_F \varepsilon_F + \mathcal{R}_F M_F^{-1} \tau_{Ff} \\ & + \mathcal{R}_F M_F^{-1} \gamma_F \tilde{\mu}_F \alpha_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \ddot{\eta}_L \\ & - (\mathcal{L}_1 \otimes I_3)^2 (\eta_F - \eta_F^*)]. \end{aligned} \quad (35)$$

Under Lemma 2, Assumption 2, and (4) and (19), we have

$$\begin{aligned} & [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)]^T (\mathcal{R}_F M_F^{-1} \tau_{Ff} - \phi_F) \\ & \leq \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\| \|\mathcal{R}_F M_F^{-1} \tau_{Ff}\| \\ & \quad - \delta \tanh\left(\frac{\delta[\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)]}{\varkappa}\right) \\ & \leq \sqrt{M} \tau_f^* / \underline{m} \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\| \\ & \quad - \delta \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\| + 0.2785 \varkappa \\ & \leq 0.2785 \varkappa \end{aligned} \quad (36)$$

for $\delta \geq \sqrt{M} \tau_f^* / \underline{m}$ and $\|\mathcal{R}_F M_F^{-1} \tau_{Ff}\| \leq \sqrt{M} \tau_f^* / \underline{m}$ with $\underline{m} = \min\{m_{il}\}$ for $i \in \mathcal{V}_F$ and $l = u, v, r$.

Substituting (36) into (35) and utilizing the property of $\|\mathcal{R}_F\| = 1$ from (4), we obtain

$$\begin{aligned} \dot{V}_2 \leq & -(\varrho_4 - \varpi) \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\|^2 \\ & + \varpi^2 \|\eta_F - \eta_F^*\| \|\mathcal{R}_F \nu_F + \nabla(\eta_F) + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L\| \\ & + \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\| \|\varepsilon_F\| \\ & + \|(\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \ddot{\eta}_L\| + \|\tilde{\Theta}_F^T \varphi_F(\nu_F)\| \\ & + [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L \\ & + \nabla(\eta_F)]^T \mathcal{R}_F M_F^{-1} \gamma_F \tilde{\mu}_F \alpha_F \end{aligned} \quad (37)$$

where $\varrho_4 = \lambda_{\min}(K_F^c)$ and $\varpi = \|\mathcal{L}_1 \otimes I_3\|_F$.

Step 3: Consider a Lyapunov function candidate $V_3 = \frac{1}{2} \text{tr}(\tilde{\mu}_F^T \gamma_F^T \Gamma_{2F}^{-1} \tilde{\mu}_F)$ with $\Gamma_{2F} = \text{diag}\{\Gamma_{i2}\}$ for $i \in \mathcal{V}_F$.

In what follows, the dynamics of V_3 from (17) is expressed as follows:

$$\begin{aligned} \dot{V}_3 = & -\text{tr}(\tilde{\mu}_F^T \gamma_F^T \beta_{2F} \tilde{\mu}_F) - \text{tr}(\tilde{\mu}_F^T \gamma_F^T M_F^{-1} \mathcal{R}_F^T [\mathcal{R}_F \nu_F \\ & + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)] \alpha_F^T) \end{aligned} \quad (38)$$

where $\beta_{2F} = \text{diag}\{\beta_{i2}\}$ for $i \in \mathcal{V}_F$.

By means of the property that $\text{tr}(ab^T) = a^T b$, one obtains

$$\begin{aligned} \dot{V}_3 \leq & -\varrho_5 \|\tilde{\mu}_F\|_F^2 + \varrho_6 \|\tilde{\mu}_F\|_F \|\mu_F\|_F - \alpha_F^T \tilde{\mu}_F^T \gamma_F^T M_F^{-1} \mathcal{R}_F^T \\ & \times [\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)] \end{aligned} \quad (39)$$

where $\varrho_5 = \min\{\gamma_{il} \beta_{i2}\}$ and $\varrho_6 = \max\{\gamma_{il} \beta_{i2}\}$ for $i \in \mathcal{V}_F$ and $l = u, v, r$.

Define $V = \sum_{i=1}^3 V_i$ and take its derivative based on above results in (32), (37), and (39) as follows:

$$\begin{aligned} \dot{V} \leq & -\varrho_3 \|\eta_F - \eta_F^*\|^2 \\ & - (\varrho_4 - \varpi) \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\|^2 \\ & - \varrho_5 \|\tilde{\mu}_F\|_F^2 + (2\lambda_{\max}(\mathbf{P}) + \varpi^2) \|\eta_F - \eta_F^*\| \\ & \times \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\| \\ & + \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\| \\ & \times \left[\|(\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \ddot{\eta}_L\| + \|\varepsilon_F\| + \|\tilde{\Theta}_F^T \varphi_F(\nu_F)\| \right] \\ & + \varrho_6 \|\tilde{\mu}_F\|_F \|\mu_F\|_F \\ = & -\mathbf{E}_2^T \mathbf{A}_2 \mathbf{E}_2 + \mathbf{B}_2^T \mathbf{E}_2 \end{aligned} \quad (40)$$

where $\mathbf{E}_2 = [\|\eta_F - \eta_F^*\|, \|\mathcal{R}_F \nu_F + (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \dot{\eta}_L + \nabla(\eta_F)\|, \|\tilde{\mu}_F\|_F]^T$, $\mathbf{B}_2 = [0, \|(\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \ddot{\eta}_L\| + \|\varepsilon_F\| + \|\tilde{\Theta}_F^T \varphi_F(\nu_F)\|, \varrho_6 \|\mu_F\|_F]^T$, and

$$\mathbf{A}_2 = \begin{bmatrix} \varrho_3 & -\lambda_{\max}(\mathbf{P}) - \frac{\varpi^2}{2} & 0 \\ -\lambda_{\max}(\mathbf{P}) - \frac{\varpi^2}{2} & \varrho_4 - \varpi & 0 \\ 0 & 0 & \varrho_5 \end{bmatrix}.$$

If $\varrho_4 > (\lambda_{\max}(\mathbf{P}) + \varpi^2)^2 / 4\varrho_3 + \varpi$, $\mathbf{A}_2$ is positive definite. Then, $\dot{V}$ satisfies

$$\dot{V} \leq -\lambda_{\min}(\mathbf{A}_2) \|\mathbf{E}_2\|^2 + \|\mathbf{B}_2\| \|\mathbf{E}_2\|. \quad (41)$$

From Lemmas 1 and 3, Assumptions 1–3, and the fact that $\|\varphi_i(\nu_i)\| \leq 1$, we obtain that $\|(\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \ddot{\eta}_L\|$ and $\|\tilde{\Theta}_F^T \varphi_F(\nu_F)\|$ are bounded, namely, $\|(\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_3) \ddot{\eta}_L\| \leq \sqrt{N - M} \eta_3^*$ and $\|\tilde{\Theta}_F^T \varphi_F(\nu_F)\| \leq \sqrt{3M} \theta^*$. Due to $0 < \gamma_{il} < 1$ and $\gamma_i \mu_i = I_3$, it renders that $\|\mu_F\|_F$ is bounded and given by $\|\mu_F\|_F \leq \sqrt{3M} / \underline{\gamma}$. Therefore, $\mathbf{B}_2$ is bounded by $\|\mathbf{B}_2\| \leq \sqrt{N - M} \eta_3^* + \sqrt{3M}(\varepsilon^* + \theta^* + 1/\underline{\gamma})$ such that

$$\dot{V} \leq -\lambda_{\min}(\mathbf{A}_2) (1 - c_2) \|\mathbf{E}_2\|^2 \quad (42)$$

for $\|\mathbf{E}_2\| \geq [\sqrt{N - M} \eta_3^* + \sqrt{3M}(\varepsilon^* + \theta^* + 1/\underline{\gamma})] / c_2 \lambda_{\min}(\mathbf{A}_2)$ with $0 < c_2 < 1$. In conclusion, the closed-loop system is input-to-state stable, and the ultimate bound is presented by

$$\begin{aligned} \|\mathbf{E}_2(t)\| \leq & \max \left\{ \mathbf{E}_2(t_0) e^{-\lambda_{\min}(\mathbf{A}_2)(1-c_2)(t-t_0)}, \right. \\ & \left. \frac{\sqrt{N - M} \eta_3^* + \sqrt{3M}(\varepsilon^* + \theta^* + 1/\underline{\gamma})}{c_2 \lambda_{\min}(\mathbf{A}_2)} \right\} \quad \forall t \geq t_0. \end{aligned} \quad (43)$$

Remark 4: Here are relevant parameter guidelines to reduce the ultimate upper bound of $\|\mathbf{E}_2\|$. From (43), we have $\lim_{t \rightarrow \infty} \|\mathbf{E}_2\| \leq [\sqrt{N - M} \eta_3^* + \sqrt{3M}(\varepsilon^* + \theta^* + 1/\underline{\gamma})] / [c_2 \lambda_{\min}(\mathbf{A}_2)]$. It is seen that the larger $\lambda_{\min}(\mathbf{A}_2)$ can reduce the ultimate upper bound of $\|\mathbf{E}_2\|$ via selecting the large parameters K_i^c and β_{i2} . Notice that larger gains will also lead to overlarge control forces and undesirable oscillations.

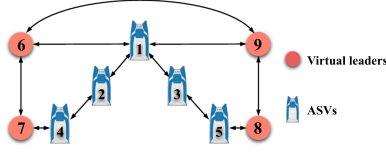


Fig. 3. Network topology for the noncooperative game.

TABLE II
PARAMETERS OF THE PROPOSED METHOD

Parameters	Values
$[\eta_1^T(0), \nu_1^T(0)]$	$[-30, 0, \pi/2, 0, 0, 0]$
$[\eta_2^T(0), \nu_2^T(0)]$	$[-30, -10, \pi/2, 0, 0, 0]$
$[\eta_3^T(0), \nu_3^T(0)]$	$[-30, 10, \pi/2, 0, 0, 0]$
$[\eta_4^T(0), \nu_4^T(0)]$	$[-40, -10, \pi/2, 0, 0, 0]$
$[\eta_5^T(0), \nu_5^T(0)]$	$[-40, 10, \pi/2, 0, 0, 0]$

Therefore, a tradeoff should be recommended between control performance and implementation requirement.

IV. SIMULATION RESULTS

This section conducts a simulation verification to demonstrate the effectiveness of the proposed noncooperative-game-based fault-tolerant containment control scheme in Fig. 2. Consider a containment pattern of five ASVs guided by four virtual leaders, namely, $\mathcal{V}_F = \{1, 2, 3, 4, 5\}$ and $\mathcal{V}_L = \{6, 7, 8, 9\}$. The information relation among nodes is depicted via a topology in Fig. 3.

This simulation employs the model ship named by CyberShip II in [62] with parameters $M_i = \text{diag}\{25.8, 33.8, 2.76\}$, $C_i = [0, 0, -33.8v_i - 1.0948r_i; 0, 0, 25.8u_i; -33.8v_i - 1.0948r_i, 25.8u_i, 0]$, and $D_i = [0.7225 + 1.3274|u_i| + 5.8664u_i^2, 0, 0; 0.8896 + 36.5|v_i| + 0.805|r_i|, 7.25 + 0.845|v_i| + 3.45|r_i|, 0; 0.0313 + 3.96|v_i| + 0.13|r_i|, 1.9 - 0.08|v_i| + 0.75|r_i|]$. The environmental disturbance τ_{iw} is modeled as a Gauss–Markov process [63] as $\dot{\tau}_{iw} + \tau_{iw} = w_i$, where $w_i \in \mathbb{R}^3$ is the white noise. This simulation is conducted in the MATLAB/Simulink environment with the simulation time of 400 s and the sampling frequency of 100 Hz. For the clarity, the initial states of ASVs are presented in Table II. The reference trajectory $\eta_{kd} = [x_{kd}, y_{kd}, \psi_{kd}]^T$ with $\psi_{kd} = \text{atan2}(\dot{y}_{kd}, \dot{x}_{kd})$ is given by $[x_{6d}, y_{6d}] = [40 - 40\sqrt{2}\sin(t/80 - 3\pi/16), 40 - 40\sqrt{2}\cos(t/80 - 3\pi/16)]$, $[x_{7d}, y_{7d}] = [40 - 40\sqrt{2}\sin((t - 40)/80 - 3\pi/16), 40 - 40\sqrt{2}\cos((t - 40)/80 - 3\pi/16)]$, $[x_{8d}, y_{8d}] = [40 - 20\sqrt{2}\sin((t - 40)/80 - 3\pi/16), 40 - 20\sqrt{2}\cos((t - 40)/80 - 3\pi/16)]$, and $[x_{9d}, y_{9d}] = [40 - 20\sqrt{2}\sin(t/80 - 3\pi/16), 40 - 20\sqrt{2}\cos(t/80 - 3\pi/16)]$. The control parameters are set as $K_i^o = \text{diag}\{2, 2, 2\}$, $K_i^c = \text{diag}\{2, 2, 2\}$, $\phi_i = 5$, $\Gamma_{i1} = 50$, $\beta_{i1} = 200$, $\beta_{i2} = 0.5$, $\Gamma_{i2} = 5$, $\delta = 20$, and $\epsilon = 2$. Moreover, the membership functions for fuzzy approximation are chosen as $\mu_{\mathcal{J}_1^l}(u_i) = e^{-[u_i - 2(l-10)]^2/4}$, $\mu_{\mathcal{J}_2^l}(v_i) = e^{-[v_i - 0.8(l-10)]^2/0.64}$, and $\mu_{\mathcal{J}_3^l}(r_i) = e^{-[r_i - 2(l-10)]^2/4}$ with $l = 1, \dots, 19$. Without loss of generality, the fault parameters

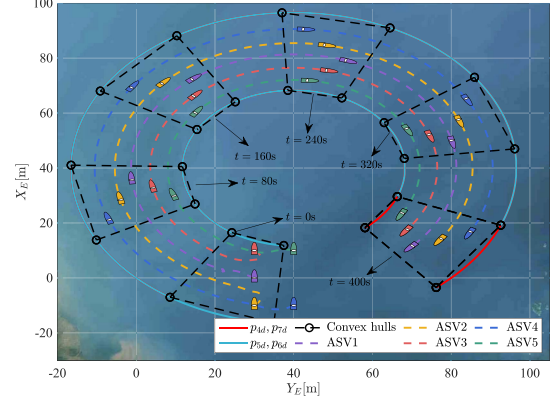
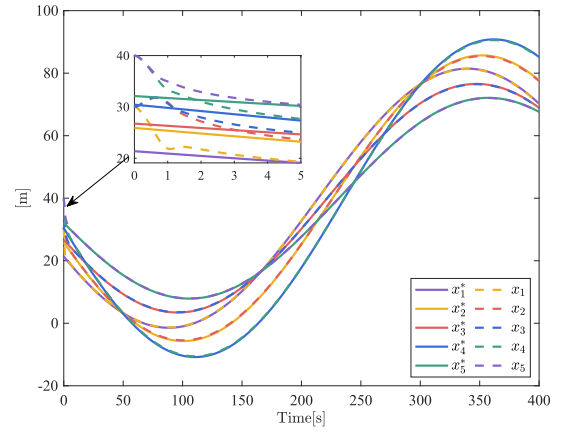


Fig. 4. Containment pattern of five ASVs based on the proposed method.

Fig. 5. Evolution of x_i and x_i^* for $i \in \mathcal{V}_F$.

are selected as $\gamma_i = \text{diag}\{0.6, 0.8, 0.7\}$ and

$$\tau_{if} = \begin{cases} [8|\sin(0.2t)|, 2, 2]^T, & 100 \leq t < 150 \\ [0, 5|\cos(0.1t)|, 5]^T, & 200 \leq t < 250 \\ [5, 0, -2|\cos(0.3t)|]^T, & 200 \leq t < 250 \\ [0, 0, 0]^T, & \text{others.} \end{cases} \quad (44)$$

To quantitatively evaluate performance metrics, we define the average position-orientation tracking error (APOTE) and the average disturbance approximation error (ADAE) as follows:

$$\text{APOTE}_i = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \|\eta_i(\xi) - \eta_i^*(\xi)\|^2 d\xi} \quad (45)$$

$$\text{ADAE}_l = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} (\hat{\sigma}_l(\xi) - \sigma_l(\xi))^2 d\xi} \quad (46)$$

where $[t_1, t_2]$ denotes some time interval of simulation run for $0 \leq t_1 < t_2$.

Simulation results based on our proposed method are plotted in Figs. 4–11, where the gray background indicates the encountering period of actuator faults in (44). Fig. 4 draws the actual trajectories of five ASVs and four virtual leaders

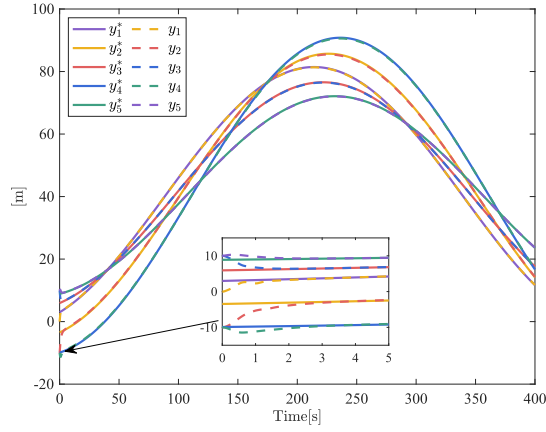
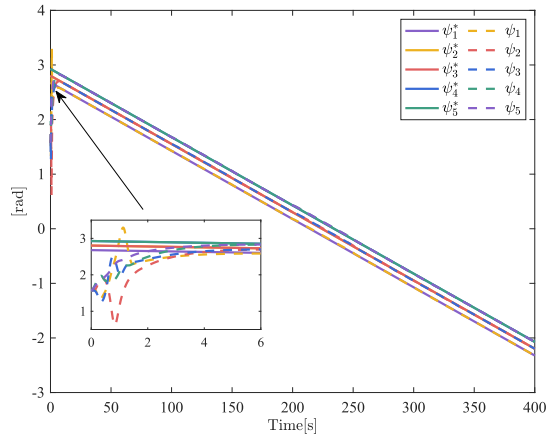
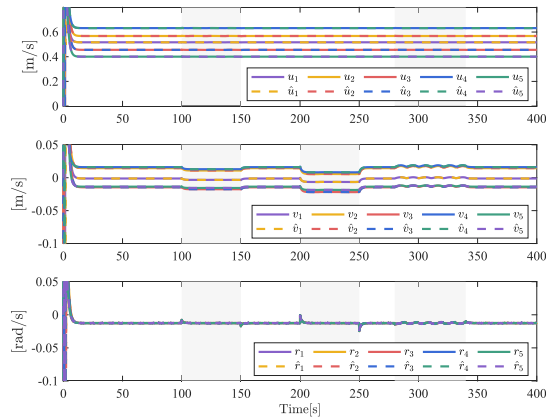
Fig. 6. Evolution of y_i and y_i^* for $i \in \mathcal{V}_F$.Fig. 7. Evolution of ψ_i and ψ_i^* for $i \in \mathcal{V}_F$.

Fig. 8. Velocity estimation profiles of the fuzzy predictor for ASV1.

with snapshots of all ASVs and virtual leaders at 0, 80, 160, 240, 320, and 400 s. It is observed that ASV1–ASV5 under the designed controller (18) accomplish the containment pattern defined in Fig. 3. Figs. 5–7 display the positions (x_i and y_i), orientations (ψ_i), and the corresponding NE curves of five ASVs. It shows that each ASV converges to the NE positions and

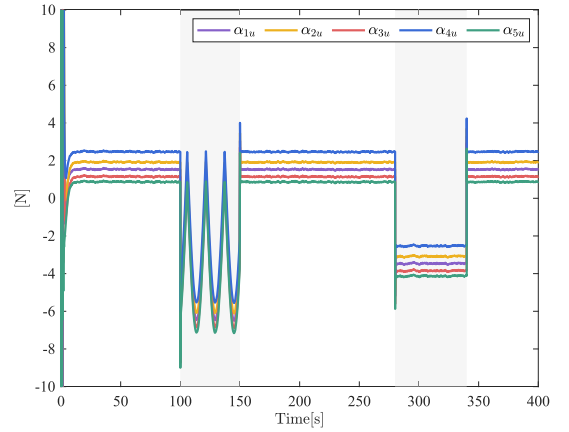


Fig. 9. Immediate control signals in the surge direction.

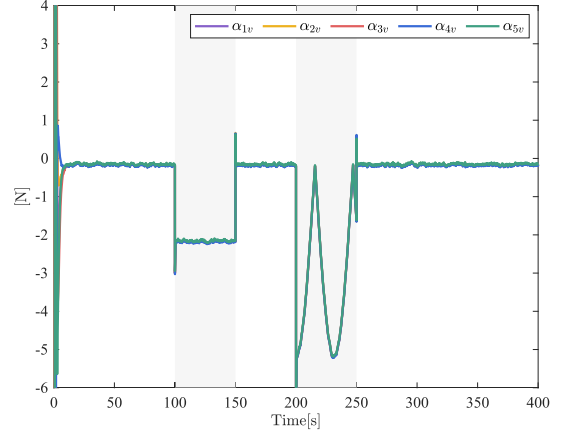


Fig. 10. Immediate control signals in the sway direction.

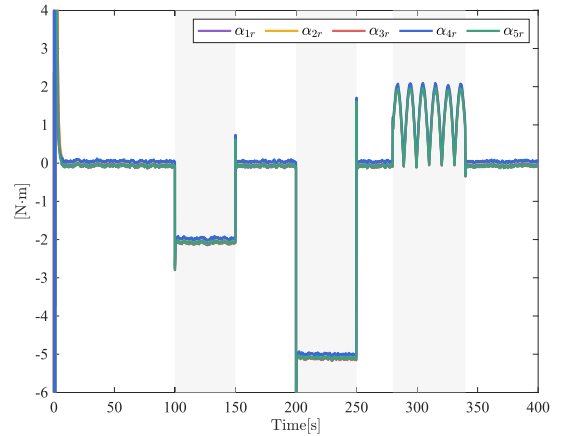


Fig. 11. Immediate control signals in the yaw direction.

orientations despite uncertainties, environmental disturbances, and actuator faults. Fig. 8 presents the actual surge, sway, and yaw speeds and the estimations from the proposed HT-based fuzzy predictor (12). Figs. 9–11 present the evolving curves of intermediate control signals in surge, sway, and yaw directions, respectively. When the actuator is encountered in the

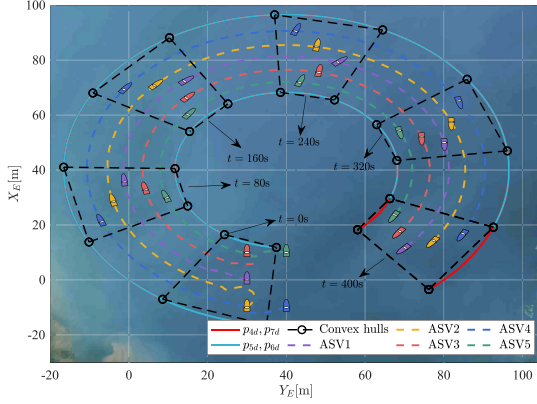


Fig. 12. Containment pattern of five ASVs based on the comparative method [19].

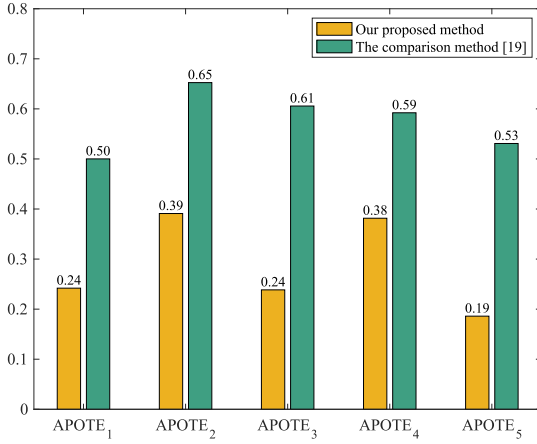


Fig. 13. APOTE of our proposed method and the comparative method [19].

fault situation, the control signals adjust to guarantee the containment tracking performance at once. For instance, the surge thrusters of ASVs experience the bias fault $8|\sin(0.2t)|$ during $t \in (100, 150)$. To avoid the negative effects, the controller α_{iu} starts to degrade under the robust term ϕ_{iu} . The reasons behind fluctuations of control signals during other faults are also similar and omitted in detail.

To further illustrate the merits of the proposed noncooperative-game-based fault-tolerant containment control method and the HT-based fuzzy predictor, we add two simulation results compared to the containment control method [19] and the fuzzy predictor [64]. To ensure the fairness, same initial conditions and parameters are adopted in the simulation comparison, which are plotted in Figs. 12–15. From Figs. 4 and 12, the multi-ASV containment pattern can be achieved via both our proposed method and the comparative method. However, from snapshots at 240 and 320 s, the negative effects of actuator faults cannot be tackled by the comparative method. Moreover, Fig. 13 presents the quantitative evaluation results defined by (46) based on our proposed method and the comparative method. It can be seen that the proposed

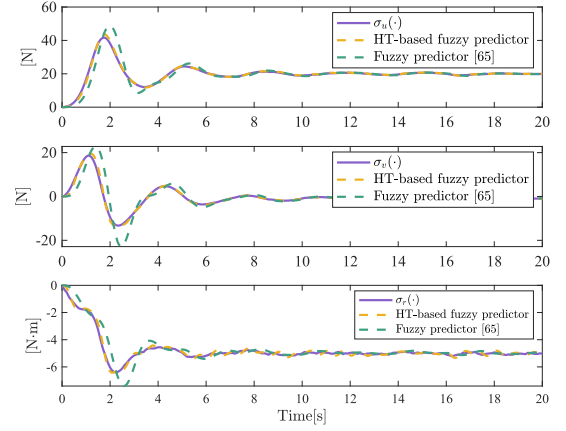


Fig. 14. Performance comparisons of the HT-based fuzzy predictor and the fuzzy predictor [64].

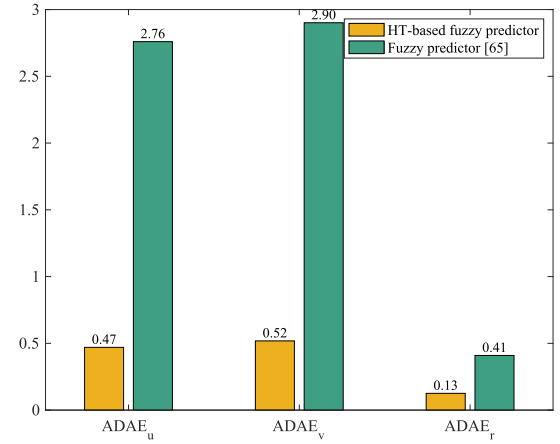


Fig. 15. ADAE of the HT-based fuzzy predictor and the fuzzy predictor [64].

method has the smaller APOTE, which indicates the better tracking performance. Figs. 14 and 15 depict the comparison results of two predictors. From Fig. 14, it is clearly observed that the HT-based fuzzy predictor can converge to the actual disturbances $\sigma_u(\cdot)$, $\sigma_v(\cdot)$, and $\sigma_r(\cdot)$ with faster speeds. Based on the performance metrics defined in (46), Fig. 14 shows that our proposed predictor displays the better approximation ability for unknown disturbances.

V. CONCLUSION

This article investigated a noncooperative-game-based fault-tolerant fuzzy containment control method for networked ASVs subject to uncertainties, environmental disturbances, and actuator faults. Specifically, an HT-based fuzzy predictor was developed to estimate lumped disturbances consisting of model uncertainties and environmental disturbances. The HT-based adaptive law can improve the convergence rate of fuzzy weight. With the designed HT-based fuzzy predictor, a noncooperative-game-based fuzzy containment control law was developed to achieve NE seeking in the presence of actuator faults. Finally,

comparison results with quantitative evaluations illustrated the effectiveness and superiority of the proposed noncooperative-game-based fault-tolerant containment control method under unknown actuator faults. For the future perspectives, it is promising to address: 1) the safety-critical optimal containment control of ASVs in an obstacle environment and 2) the potential game-based optimal containment control of ASVs suffering from cooperation constraints.

REFERENCES

- [1] Y. Shi, C. Shen, H. Fang, and H. Li, "Advanced control in marine mechatronic systems: A survey," *IEEE/ASME Trans. Mechatron.*, vol. 22, no. 3, pp. 1121–1131, Jun. 2017.
- [2] Z. Peng, J. Wang, D. Wang, and Q.-L. Han, "An overview of recent advances in coordinated control of multiple autonomous surface vehicles," *IEEE Trans. Ind. Informat.*, vol. 17, no. 2, pp. 732–745, Feb. 2021.
- [3] J. Li, G. Zhang, C. Liu, and W. Zhang, "COLREGs-constrained adaptive fuzzy event-triggered control for underactuated surface vessels with the actuator failures," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 12, pp. 3822–3832, Dec. 2021.
- [4] X. Wang, S. Li, and P. Shi, "Distributed finite-time containment control for double-integrator multiagent systems," *IEEE Trans. Cybern.*, vol. 44, no. 9, pp. 1518–1528, Sep. 2014.
- [5] G. Wen, Y. Zhao, Z. Duan, W. Yu, and G. Chen, "Containment of higher-order multi-leader multi-agent systems: A dynamic output approach," *IEEE Trans. Autom. Control*, vol. 61, no. 4, pp. 1135–1140, Apr. 2016.
- [6] W. Wu and S. Tong, "Fixed-time adaptive fuzzy containment dynamic surface control for nonlinear multiagent systems," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 12, pp. 5237–5248, Dec. 2022.
- [7] W. Wu, Y. Zhang, Z. Li, J.-G. Lu, and W. Zhang, "Constrained safe cooperative maneuvering of autonomous surface vehicles: A control barrier function approach," *IEEE Trans. Syst., Man, Cybern. Syst.*, early access, Jan 25, 2024, doi: [10.1109/TSMC.2023.3345847](https://doi.org/10.1109/TSMC.2023.3345847).
- [8] J. Yu, W. Xiao, X. Dong, Q. Li, and Z. Ren, "Practical formation-containment tracking for multiple autonomous surface vessels system," *IET Control. Theory Appl.*, vol. 13, no. 17, pp. 2894–2905, 2019.
- [9] Z. Peng, J. Wang, and D. Wang, "Containment maneuvering of marine surface vehicles with multiple parameterized paths via spatial-temporal decoupling," *IEEE/ASME Trans. Mechatron.*, vol. 22, no. 2, pp. 1026–1036, Apr. 2017.
- [10] Z. Peng, J. Wang, and D. Wang, "Distributed containment maneuvering of multiple marine vessels via neurodynamics-based output feedback," *IEEE Trans. Ind. Electron.*, vol. 64, no. 5, pp. 3831–3839, May 2017.
- [11] J. Ghommam, M. Saad, and F. Mnif, "Prescribed performances based fuzzy-adaptive output feedback containment control for multiple underactuated surface vessels," *Ocean Eng.*, vol. 249, 2022, Art. no. 110898.
- [12] Y. Qu, W. Zhao, Z. Yu, and B. Xiao, "Distributed prescribed performance containment control for unmanned surface vehicles based on disturbance observer," *ISA Trans.*, vol. 125, pp. 699–706, 2022.
- [13] Q. Deng, Y. Peng, D. Qu, T. Han, and X. Zhan, "Neuro-adaptive containment control of unmanned surface vehicles with disturbance observer and collision-free," *ISA Trans.*, vol. 129, pp. 150–156, 2022.
- [14] N. Gu, D. Wang, Z. Peng, and L. Liu, "Observer-based finite-time control for distributed path maneuvering of underactuated unmanned surface vehicles with collision avoidance and connectivity preservation," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 51, no. 8, pp. 5105–5115, Aug. 2021.
- [15] W. Wu, Y. Zhang, W. Zhang, and W. Xie, "Output-feedback finite-time safety-critical coordinated control of path-guided marine surface vessels based on neurodynamic optimization," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 53, no. 3, pp. 1788–1800, Mar. 2023.
- [16] Z.-W. Liu, H. Hou, and Y.-W. Wang, "Formation-containment control of multiple underactuated surface vessels with sampling communication via hierarchical sliding mode approach," *ISA Trans.*, vol. 124, pp. 458–467, 2022.
- [17] Y. Zhang, D. Wang, Y. Yin, and Z. Peng, "Event-triggered distributed coordinated control of networked autonomous surface vehicles subject to fully unknown kinetics via concurrent-learning-based neural predictor," *Ocean Eng.*, vol. 234, Aug. 2021, Art. no. 108966.
- [18] M. Li, C. Guo, H. Yu, and Y. Yuan, "Event-triggered containment control of networked underactuated unmanned surface vehicles with finite-time convergence," *Ocean Eng.*, vol. 246, 2022, Art. no. 110548.
- [19] H. Wang, H. Tan, and Z. Peng, "Quantized communications in containment maneuvering for output constrained marine surface vehicles: Theory and experiment," *IEEE Trans. Ind. Electron.*, vol. 71, no. 1, pp. 880–889, Jan. 2024.
- [20] W. Wu, Z. Peng, D. Wang, L. Liu, and Q.-L. Han, "Network-based line-of-sight path tracking of underactuated unmanned surface vehicles with experiment results," *IEEE Trans. Cybern.*, vol. 52, no. 10, pp. 10937–10947, Oct. 2022.
- [21] M. Ye and G. Hu, "Distributed seeking of time-varying Nash equilibrium for non-cooperative games," *IEEE Trans. Autom. Control*, vol. 60, no. 11, pp. 3000–3005, Nov. 2015.
- [22] M. Ye, J. Yin, and L. Yin, "Distributed Nash equilibrium seeking for games in second-order systems without velocity measurement," *IEEE Trans. Autom. Control*, vol. 67, no. 11, pp. 6195–6202, Nov. 2022.
- [23] Z. Feng, G. Hu, X. Dong, and J. Lu, "Adaptively distributed Nash equilibrium seeking of noncooperative games for uncertain heterogeneous linear multi-agent systems," *IEEE Trans. Netw. Sci. Eng.*, vol. 10, no. 6, pp. 3871–3882, Nov./Dec. 2023, doi: [10.1109/TNSE.2023.3275326](https://doi.org/10.1109/TNSE.2023.3275326).
- [24] X. He and J. Huang, "Distributed Nash equilibrium seeking with dynamics subject to disturbance of unknown frequencies over jointly strongly connected switching networks," *IEEE Trans. Autom. Control*, vol. 69, no. 1, pp. 606–613, Jan. 2024, doi: [10.1109/TAC.2023.3271791](https://doi.org/10.1109/TAC.2023.3271791).
- [25] X. Fang, J. Zhou, and G. Wen, "Location game of multiple unmanned surface vessels with quantized communications," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 69, no. 3, pp. 1322–1326, Mar. 2022.
- [26] X. Fang, G. Wen, X. Yu, and G. Chen, "Formation control for unmanned surface vessels: A game-theoretic approach," *Asian J. Control*, vol. 24, no. 2, pp. 498–509, 2022.
- [27] J. Shi and M. Ye, "Distributed optimal formation control for unmanned surface vessels by a regularized game-based approach," *IEEE/CAA J. Autom. Sinica*, vol. 11, no. 1, pp. 276–278, Jan. 2024.
- [28] G. Wen, X. Fang, J. Zhou, and J. Zhou, "Robust formation tracking of multiple autonomous surface vessels with individual objectives: A noncooperative game-based approach," *Control Eng. Pract.*, vol. 119, 2022, Art. no. 104975.
- [29] Y. Zhang, W. Wu, and W. Zhang, "Noncooperative game-based cooperative maneuvering of intelligent surface vehicles via accelerated learning-based neural predictors," *IEEE Trans. Intell. Veh.*, vol. 8, no. 3, pp. 2212–2221, Mar. 2023.
- [30] K. Yu, Y. Li, M. Lv, and S. Tong, "Distributed optimal formation control of multiple unmanned surface vehicles with Stackelberg differential graphical game," *IEEE Trans. Artif. Intell.*, early access, doi: [10.1109/TAI.2024.3357795](https://doi.org/10.1109/TAI.2024.3357795).
- [31] X. Guo, R. Cui, and W. Yan, "Pursuit-evasion games of marine surface vessels using neural network-based control," *IEEE Trans. Syst., Man, Cybern. Syst.*, early access, doi: [10.1109/TSMC.2023.3347044](https://doi.org/10.1109/TSMC.2023.3347044).
- [32] X. Jin, "Fault tolerant finite-time leader-follower formation control for autonomous surface vessels with LOS range and angle constraints," *Automatica*, vol. 68, pp. 228–236, 2016.
- [33] L.-Y. Hao, H. Zhang, H. Li, and T.-S. Li, "Sliding mode fault-tolerant control for unmanned marine vehicles with signal quantization and time-delay," *Ocean Eng.*, vol. 215, 2020, Art. no. 107882.
- [34] N. Wang and Z. Deng, "Finite-time fault estimator based fault-tolerance control for a surface vehicle with input saturations," *IEEE Trans. Ind. Inform.*, vol. 16, no. 2, pp. 1172–1181, 2020.
- [35] Z.-Q. Liu, Y.-L. Wang, and Q.-L. Han, "Adaptive fault-tolerant trajectory tracking control of twin-propeller non-rudder unmanned surface vehicles," *Ocean Eng.*, vol. 285, 2023, Art. no. 115294.
- [36] X. Meng, G. Zhang, and B. Han, "Fault-tolerant control of underactuated MSVs based on neural finite-time disturbance observer: An event-triggered mechanism," *J. Franklin Inst.*, vol. 361, no. 4, 2024, Art. no. 106603.
- [37] Y. Lu, C. Wen, T. Shen, and W. Zhang, "Bearing-based adaptive neural formation scaling control for autonomous surface vehicles with uncertainties and input saturation," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 10, pp. 4653–4664, Oct. 2021.
- [38] T. Li, W. Bai, Q. Liu, Y. Long, and C. L. P. Chen, "Distributed fault-tolerant containment control protocols for the discrete-time multiagent systems via reinforcement learning method," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 34, no. 8, pp. 3979–3991, Aug. 2023.

- [39] L. Liu, D. Wang, Z. Peng, and Q.-L. Han, "Distributed path following of multiple under-actuated autonomous surface vehicles based on data-driven neural predictors via integral concurrent learning," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 12, pp. 5334–5344, Dec. 2021.
- [40] W. Wu, Y. Zhang, W. Zhang, and W. Xie, "Distributed finite-time performance-prescribed time-varying formation control of autonomous surface vehicles with saturated inputs," *Ocean Eng.*, vol. 266, 2022, Art. no. 112866, doi: [10.1016/j.oceaneng.2022.112866](https://doi.org/10.1016/j.oceaneng.2022.112866).
- [41] Z. Peng, Y. Jiang, L. Liu, and Y. Shi, "Path-guided model-free flocking control of unmanned surface vehicles based on concurrent learning extended state observers," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 53, no. 8, pp. 4729–4739, Aug. 2023.
- [42] Q. Liu, Y. Long, T. Li, J. H. Park, and C. L. P. Chen, "Fault detection for unmanned marine vehicles under replay attack," *IEEE Trans. Fuzzy Syst.*, vol. 31, no. 5, pp. 1716–1728, May 2023.
- [43] Y. Jiang, Z. Peng, L. Liu, D. Wang, and F. Zhang, "Safety-critical co-operative target enclosing control of autonomous surface vehicles based on finite-time fuzzy predictors and input-to-state safe high order control barrier functions," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 3, pp. 816–830, Mar. 2024.
- [44] W. Wu, R. Ji, W. Zhang, and Y. Zhang, "Transient-reinforced tunnel coordinated control of underactuated marine surface vehicles with actuator faults," *IEEE Trans. Intell. Transp. Syst.*, vol. 25, no. 2, pp. 1872–1881, Feb. 2024.
- [45] Y. Zhang, W. Wu, J. Lu, and W. Zhang, "Neural predictor-based dynamic surface parallel control for MIMO uncertain nonlinear strict-feedback systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 70, no. 8, pp. 2909–2913, Aug. 2023.
- [46] J. Mei, W. Ren, and G. Ma, "Distributed containment control for Lagrangian networks with parametric uncertainties under a directed graph," *Automatica*, vol. 48, no. 4, pp. 653–659, 2012.
- [47] X. Dong, B. Yu, Z. Shi, and Y. Zhong, "Time-varying formation control for unmanned aerial vehicles: Theories and applications," *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 1, pp. 340–348, Jan. 2015.
- [48] Y. Yu, J. Guo, M. Chadli, and Z. Xiang, "Distributed adaptive fuzzy formation control of uncertain multiple unmanned aerial vehicles with actuator faults and switching topologies," *IEEE Trans. Fuzzy Syst.*, vol. 31, no. 3, pp. 919–929, Mar. 2023.
- [49] S. He, S.-L. Dai, Z. Zhao, T. Zou, and Y. Ma, "UDE-based distributed formation control for MSVs with collision avoidance and connectivity preservation," *IEEE Trans. Ind. Inform.*, vol. 20, no. 2, pp. 1476–1487, Feb. 2024.
- [50] L. Wang, D. Zhu, W. Pang, and C. Luo, "A novel obstacle avoidance consensus control for multi-AUV formation system," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 5, pp. 1304–1318, May 2023.
- [51] Z.-Q. Liu, X. Ge, H. Xie, Q.-L. Han, J. Zheng, and Y.-L. Wang, "Secure leader–follower formation control of networked mobile robots under replay attacks," *IEEE Trans. Ind. Inform.*, vol. 20, no. 3, pp. 4149–4159, Mar. 2024.
- [52] L.-X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*. Upper Saddle River, NJ, USA: Prentice-Hall, 1994.
- [53] T. I. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control*. Hoboken, NJ, USA: Wiley, 2011.
- [54] K. Li, K. Feng, and Y. Li, "Fuzzy adaptive fault-tolerant formation control for USVs with intermittent actuator faults," *IEEE Trans. Intell. Veh.*, vol. 9, no. 3, pp. 4445–4455, Mar. 2024.
- [55] N. Gu, D. Wang, Z. Peng, T. Li, and S. Tong, "Model-free containment control of underactuated surface vessels under switching topologies based on guiding vector fields and data-driven neural predictors," *IEEE Trans. Cybern.*, vol. 52, no. 10, pp. 10843–10854, Oct. 2022.
- [56] W. Wu, Z. Peng, L. Liu, and D. Wang, "A general safety-certified cooperative control architecture for interconnected intelligent surface vehicles with applications to vessel train," *IEEE Trans. Intell. Veh.*, vol. 7, no. 3, pp. 627–637, Sep. 2022.
- [57] X. Xu, Y. Lu, X. Liu, and W. Zhang, "Intelligent collision avoidance algorithms for USVs via deep reinforcement learning under COLREGs," *Ocean Eng.*, vol. 217, 2020, Art. no. 107704.
- [58] W. Wu and S. Tong, "Collision-free finite-time adaptive fuzzy output-feedback formation control for unmanned surface vehicle systems," *IEEE Trans. Intell. Veh.*, vol. 9, no. 1, pp. 1094–1103, Jan. 2024.
- [59] J. E. Gaudio, A. M. Annaswamy, M. A. Bolender, E. Lavretsky, and T. E. Gibson, "A class of high order tuners for adaptive systems," *IEEE Control Syst. Lett.*, vol. 5, no. 2, pp. 391–396, Apr. 2021.
- [60] J. E. Gaudio, A. M. Annaswamy, J. M. Moreu, M. A. Bolender, and T. E. Gibson, "Accelerated learning with robustness to adversarial regressors," in *Proc. 3rd Conf. Learn. Dyn. Control*, 2021, pp. 636–650.
- [61] A. M. Annaswamy, A. Guha, Y. Cui, S. Tang, P. A. Fisher, and J. E. Gaudio, "Integration of adaptive control and reinforcement learning for real-time control and learning," *IEEE Trans. Autom. Control*, vol. 68, no. 12, pp. 7740–7755, Dec. 2023, doi: [10.1109/TAC.2023.3290037](https://doi.org/10.1109/TAC.2023.3290037).
- [62] R. Skjetne, T. I. Fossen, and P. V. Kokotović, "Adaptive maneuvering, with experiments, for a model ship in a marine control laboratory," *Automatica*, vol. 41, no. 2, pp. 289–298, Feb. 2005.
- [63] D. Roetenberg, H. J. Luinge, C. T. Baten, and P. H. Veltink, "Compensation of magnetic disturbances improves inertial and magnetic sensing of human body segment orientation," *IEEE Trans. Neural Syst. Rehabil. Eng.*, vol. 13, no. 3, pp. 395–405, Sep. 2005.
- [64] Y. Jiang, Z. Peng, D. Wang, Y. Yin, and Q.-L. Han, "Cooperative target enclosing of ring-networked under-actuated autonomous surface vehicles based on data-driven fuzzy predictors and extended state observers," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 7, pp. 2515–2528, Jul. 2022.



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