Finite-time Extended State Observer-Based Performance-Critical Control for Uncertain MIMO Nonlinear Systems

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Abstract. This paper studies a prescribed performance control (PPC) problem of uncertain nonlinear systems in a multi-input multi-output Brunovsky form. An observer-based performance-critical control method is proposed, which is different from existing PPC methods using error transformation functions (ETFs). At first, a finite-time extended state observer (FTESO) is designed to identify the nonlinear term. Then, based on the proposed FTESO, a nominal integral sliding mode controller is devised without considering user-specified performance indexes. Next, we construct the input-to-state safe high-order control barrier functions (ISSf-HOCBFs) for tracking error systems with unknown disturbances. The performance-critical input constraints are devised by using PPC constraint-based ISSf-HOCBFs. Later, a modified controller is solved by the quadratic program unifying nominal controller with performance-critical input constraints. Finally, it is proved that the closed-loop system is input-to-state safe by forward invariance analysis, and tracking errors evolve within prescribed constraints. Finally, a simulation example of an unmanned surface vehicle is conducted to demonstrate the effectiveness of the proposed observer-based performance-critical control method.

Keywords: Uncertain multi-input multi-output uncertain nonlinear systems, finite-time state observer, control barrier function, and prescribed performance control.

1 INTRODUCTION

In recent years, there has been an increasing focus on the control problem of uncertain nonlinear systems in multi-input multi-output (MIMO) Brunovsky form, such as unmanned aerial/surface/underwater vehicles [1–4], robots [5], and manipulators [6], etc. Due to the extensive range of applications, improving the control performance has sparked intense interest among both researchers and communities. In [7], authors firstly proposed the prescribed performance control

(PPC) technique, which allows the tracking error to evolve under a predefined convergence rate and maximum overshoot. The PPC methodology can explicitly preset the transient and steady-state indices according to specific operating scenarios. For nonlinear systems subject to internal uncertainties and external disturbances, there exist some PPC schemes using approximation tools including neural networks [8,9], fuzzy logic systems [10,11], and observers [12,13], etc. In [14], a command filter-based adaptive constrained tracking controller is developed to guarantee predefined performance under bounded forces. In [5], a vision-based fixed-time PPC formation controller is presented for wheeled robots with local relative distance and bearing angle. In [8], an experience-based PPC formation controller is developed for underactuated vessels by using a cooperative deterministic learning protocol. Consider the quantization property under a band-limited network in [15,16]. For an input-quantized strict-feedback system, [15] designs an improved performance function to achieve finite-time error convergence without finite-time control protocol. [16] presents a robust performance-adjusted trajectory tracking method for state-quantized vessels without the aid of prior knowledge and identified information. It is observed that the PPC schemes of reviewed works above are implemented based on error transformation functions (ETFs), which map the original system into an equivalent unconstrained one.

More recently, control barrier functions (CBFs) and high order control barrier functions (HOCBFs) are increasingly applied in safety-critical control fields, such as adaptive cruise control [17], lane keeping [18], and collision-free formation [19–22]. The Safety-critical controller is prior to ensuring invariances of a given set by solving a quadratic program to unify CBFs or HOCBFs with control Lyapunov functions or performance/stability-based controllers. Further, [23] proposes an input-to-state safe (ISSf) set for nonlinear systems in the presence of input disturbances, and an ISSf-CBF is defined for the single affine system, which has been used to guarantee the formation safety of multiple marine vehicles [24].

Motivated by above discussions, an observer-based performance-critical control method is developed. The main features include two folds: i) A finite-time extended state observer (FTESO) is constructed to estimate unknown uncertainties within a finite time. Based on estimated terms, a nominal integral sliding mode controller is designed to achieve the tracking objective. ii) We develop the input-to-state safe high-order HOCBFs (ISSf-HOCBFs) for error systems with unknown disturbances. Then, a performance-critical controller is presented to ensure ISSf of sets defined by prescribed performance constraints.

2 PRELIMINARIES AND PROBLEM FORMULATION

2.1 Notations

$\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{R}^n$, and $\mathbb{R}_+^{m\times m}$ represent a real constant set, a positive constant set, a real n dimensional vector set, and a positive definite $m \times m$ dimensional matrix set, respectively. $\mathbb{I}_{1:n}$ denotes a number set with $\mathbb{I}_{1:n} = \{1, ..., n\}$. diag$\{\cdot\}$ is a block-diagonal matrix. col$\{\cdot\}$ denotes a column vector. A function $\lceil x \rfloor^a$ is defined as

$\lceil x \rfloor^a = |x|^a \operatorname{sign}(x), 0 < a < 1$. $\partial \mathcal{C}$ and $\operatorname{Int}(\mathcal{C})$ denote the boundary and interior of a set $\mathcal{C}$, respectively. Considering a system $\dot{x} = f(x) + g(x)u$, the Lie derivatives of a function $h(x)$ is defined as $L_f h(x) = \frac{\partial h(x)}{\partial x} f(x)$ and $L_g h(x) = \frac{\partial h(x)}{\partial x} g(x)$.

For a strictly increasing continuous function $\alpha(\cdot) : (-b, a) \mapsto (-\infty, \infty)$, it is an extended class $\mathcal{K}$ function denoted as $\mathcal{K}_e$ if $a, b \in \mathbb{R}^+$ and $\alpha(0) = 0$. For a strictly increasing continuous function $\alpha(\cdot) : (-b, a) \mapsto (-\infty, \infty)$, it is an extended class $\mathcal{K}_\infty$ function denoted as $\mathcal{K}_{\infty,e}$ if $a, b = \infty$ and $\alpha(0) = 0$.

2.2 Preliminaries

Definition 1: (*Relative degree* [25]) The relative degree of a continuously differentiable function $h(x) : \mathbb{R}^n \to \mathbb{R}$ for system $\dot{x} = f(x) + g(x)u$ is the number of times that $h(x)$ needs to be differentiated along the system dynamics before the control input u appears explicitly.

Lemma 1: ([25]) Assume $\phi(t) : [t_0, t_f] \to \mathbb{R}$ is a continuously differentiable function with initial time t_0 and final time t_f. If $\dot{\phi}(t) \geq \alpha(\phi(t))$ for all $t \in [t_0, t_f]$, where α is a class $\mathcal{K}$ function, and $\phi(t_0) \geq 0$, then $\phi(t) \geq 0$ for all $t \in [t_0, t_f]$.

2.3 Problem Formulation

Consider a class of uncertain nonlinear systems in MIMO Brunovsky form [26]

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = \zeta(\underline{x}) + u(t), \\ y(t) = x_1(t), \end{cases} \tag{1}$$

where $x_1 = \operatorname{col}(x_{1j})$, $x_2 = \operatorname{col}(x_{2j})$, $u = \operatorname{col}(u_j)$, and $y = \operatorname{col}(y_j)$, $j \in \mathbb{I}_{1:m}$ are the system states, the control input, the system output, respectively; $\zeta(\underline{x}) : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$ denotes unknown nonlinear smooth function with $\underline{x} = [x_1^T, x_2^T]^T$. It is assumed that its derivative $\dot{\zeta}$ is bounded with $\|\dot{\zeta}\| < \zeta^* \in \mathbb{R}^+$.

Define the tracking errors $e_1 = \operatorname{col}(e_{1j})$ and $e_2 = \operatorname{col}(e_{2j})$, $j \in \mathbb{I}_{1:m}$ as

$$e_1(t) = x_1(t) - x_{1d}(t) - \Delta_1, \; e_2(t) = x_2(t) - x_{2d}(t) - \Delta_2. \tag{2}$$

where $\Delta_1 \in \mathbb{R}^m$ and $\Delta_2 \in \mathbb{R}^m$ are given deviations; $x_{1d} \in \mathbb{R}^m$ and $x_{2d} \in \mathbb{R}^m$ represent the desired states, which are updated by the following dynamic system

$$\begin{cases} \dot{x}_{1d}(t) = x_{2d}(t), \\ \dot{x}_{2d}(t) = u_d(\underline{x}_d), \end{cases} \tag{3}$$

where $u_d(\underline{x}_d) \in \mathbb{R}^m$ is a given continuous function with $\underline{x}_d = [x_{1d}^T, x_{2d}^T]^T$.

To obtain the user-specified tracking performance, we usually force the system output y to track the desired output signal x_{1d} such that

$$\begin{cases} e_{1j} \leq (\delta_{jr} + \lceil e_{1j,0} \rfloor^0)\rho_j(t) - \rho_{j,\infty} \lceil e_{1j,0} \rfloor^0, \\ e_{1j} \geq -(\delta_{jl} - \lceil e_{1j,0} \rfloor^0)\rho_j(t) - \rho_{j,\infty} \lceil e_{1j,0} \rfloor^0, \end{cases} \tag{4}$$

where $e_1 = \mathrm{col}(e_{1j}), j \in \mathbb{I}_{1:m}$; $0 \le \delta_{jr}, \delta_{jl} \le 1$; $e_{1j,0} = e_{1j}(t_0)$; $\rho_j(t) : (t_0, \infty) \to (\rho_{j,0}, \rho_{j,\infty})$ is a monotonically decreasing differentiable function as

$$\rho_j(t) = (\rho_{j,0} - \rho_{j,\infty})e^{-\sigma_j(t-t_0)} + \rho_{j,\infty}, \tag{5}$$

where $\sigma_j \in \mathbb{R}^+$ is a constant for designing convergence speed; $\rho_{j,0} \in \mathbb{R}^+$ and $\rho_{j,\infty} \in \mathbb{R}^+$ are user-specified parameters satisfying $\rho_{j,0} > \rho_{j,\infty}$.

Control objectives: In this paper, we aim to develop an observer-based performance-critical tracking control method for a second-order MIMO Brunovsky system subject to unknown internal uncertainties. The detailed objectives include two aspects: 1) $\zeta(\underline{x})$ can be estimated by using the proposed FTESO within a finite time; 2) e_1 and e_2 can converge and evolve within the prescribed performance constraints by using the proposed observer-based performance-critical tracking control method.

3 CONTROLLER DESIGN AND ANALYSIS

3.1 Finite-Time Extended State Observer

Before designing the performance-critical controller, we design an FTESO to estimate unknown terms as follows

$$\begin{cases} \dot{\hat{x}}_2 = -L_1^o \lceil \hat{x}_2 - x_2 \rfloor^{\frac{1}{2}} + \hat{\zeta} + u, \\ \dot{\hat{\zeta}} = -L_2^o \lceil \hat{x}_2 - x_2 \rfloor^0, \end{cases} \tag{6}$$

where $\hat{x}_2 \in \mathbb{R}^m$ and $\hat{\zeta} \in \mathbb{R}^m$ represent observations of x_2 and $\zeta(\underline{x})$, respectively; $L_1^o \in \mathbb{R}_+^{m\times m}$ and $L_2^o \in \mathbb{R}_+^{m\times m}$ are designed matrices.

Letting $\tilde{x}_2 = \hat{x}_2 - x_2 \in \mathbb{R}^m$ and $\tilde{\zeta} = \hat{\zeta} - \zeta \in \mathbb{R}^m$, it yields the error dynamics of FTESO as

$$\begin{cases} \dot{\tilde{x}}_2 = -L_1^o \lceil \tilde{x}_2 \rfloor^{\frac{1}{2}} + \tilde{\zeta}, \\ \dot{\tilde{\zeta}} = -L_2^o \lceil \tilde{x}_2 \rfloor^0 - \dot{\zeta}. \end{cases} \tag{7}$$

Under $\|\dot{\zeta}\| < \zeta^*$, it gets that the finite-time convergence of FTESO error dynamics from [27, Theorem 6]. Thus, it is concluded that the observation error $\tilde{\zeta}$ is bounded with $\|\tilde{\zeta}\| \le \iota \in \mathbb{R}^+$.

3.2 Design and Analysis of Controller

Taking the derivative of (2) along (1) and (3), and substituting $\hat{\zeta}$ into its derivative, one yields that

$$\dot{e}_1 = e_2,\ \dot{e}_2 = u - u_d + \hat{\zeta} - \tilde{\zeta}. \tag{8}$$

Then, an integral sliding surface $z \in \mathbb{R}^m$ is designed as follows

$$z = e_2 + \int_{t_0}^{t} (L_1^z e_1(\theta) + L_2^z e_2(\theta)) d\theta, \tag{9}$$

where $L_1^z \in \mathbb{R}_+^{m\times m}$ and $L_2^z \in \mathbb{R}_+^{m\times m}$ are designed parameters matrices.

Differentiating z yields that

$$\dot{z} = \dot{e}_2 + L_1^z e_1(t) + L_2^z e_2(t). \tag{10}$$

Combining (13) with (9), a nominal controller is derived as follows

$$u_{\rm nom} = -L^c z - L_1^z e_1 - L_2^z e_2 - \hat{\zeta} + u_d \tag{11}$$

where $L^c \in \mathbb{R}_+^{m\times m}$ is a gain matrix.

Substituting (11) into (10), the closed-loop system is written as

$$\dot{z} = -L^c z - \tilde{\zeta} + q. \tag{12}$$

where $q = u - u_{\rm nom}$. The following lemma presents the stability of system (12).

Lemma 2: The system (12) with the state z and inputs $\tilde{\zeta}$ and q is input-to-state stable.

Proof: Construct the following Lyapunov function $V = z^T z/2$. Taking the time derivative of V with (12), it follows that $\dot{V} = z^T(-L^c z - \tilde{\zeta} + q)$.

It renders that $\dot{V} \le -\lambda_{\min}(L^c)\|z\|^2 + \|z\|(\|\tilde{\zeta}\| + \|q\|)$, where $\lambda_{\min}(L^c)$ is the minimum eigenvalue of L^c. Since $\|z\| \ge (\|\tilde{\zeta}\| + \|q\|)/\epsilon\lambda_{\min}(L^c)$ with $\epsilon \in (0,1)$, it renders $\dot{V} \le -(1-\epsilon)\lambda_{\min}(L^c)\|z\|^2$. From [25, Lemma 4.6], it concludes that system (10) is input-to-state stable, and the ultimate bound is expressed as $z(t) \le \max\{\|z(t_0)\|e^{-(1-\epsilon)\lambda_{\min}(L^c)(t-t_0)}, (\|\tilde{\zeta}\| + \|q\|)/\epsilon\lambda_{\min}(L^c)\}$.

After above design and analysis, a nominal controller has been devised to stabilize (8), which can be viewed as a system with states e_1, e_2, the input u with disturbance $\tilde{\zeta}$. For the sake of designing performance-critical conditions, dynamics (8) is usually rewritten as follows

$$\underbrace{\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\end{bmatrix}}_{\dot{e}\in\mathbb{R}^{2m}} = \underbrace{\begin{bmatrix}e_2\\ \hat{\zeta} - u_d\end{bmatrix}}_{f\in\mathbb{R}^{2m}} + \underbrace{\begin{bmatrix}0_m\\ I_m\end{bmatrix}}_{g\in\mathbb{R}^{2m\times m}}(u+w), \tag{13}$$

where $w = -\tilde{\zeta} \in \mathbb{R}^m$. From the bounded errors $\tilde{\zeta}$, it follows that $\|w\|_\infty \le \iota$ with $\|w\|_\infty \triangleq \operatorname{ess\,sup}_{t\ge t_0}\|w(t)\|$ being the $\mathbb{L}_\infty^m$ norm.

The inequality constraints (4) can be described by using the set $\mathcal{C}_{jk}(e_j)_{,k=r,l}$ with $e_j = {\rm col}(e_{1j}, e_{2j})$. Then, the set $\mathcal{C}_{jk}$ is expressed as the following form

$$\begin{aligned}\mathcal{C}_{jk} &= \{e_j \in \mathbb{R}^2 \mid h_{jk}(e_j) \ge 0\},\\ \partial\mathcal{C}_{jk} &= \{e_j \in \mathbb{R}^2 \mid h_{jk}(e_j) = 0\},\\ {\rm Int}(\mathcal{C}_{jk}) &= \{e_j \in \mathbb{R}^2 \mid h_{jk}(e_j) > 0\},\end{aligned} \tag{14}$$

where $h_{jk}(e_j)$ is a continuously differentiable function built by (4) as follows

$$\begin{cases} h_{jr}(e_j) = (\delta_{jr} + \lceil e_{1j,0}\rfloor^0)\rho_j(t) - \rho_{j,\infty}\lceil e_{1j,0}\rfloor^0 - e_{1j},\\ h_{jl}(e_j) = (\delta_{jl} - \lceil e_{1j,0}\rfloor^0)\rho_j(t) + \rho_{j,\infty}\lceil e_{1j,0}\rfloor^0 + e_{1j}.\end{cases} \tag{15}$$

If constraints (4) hold for $\forall t \geq t_0$, it means that e_j always belongs to the sets $\mathcal{C}_{jk}(e_j)_{,k=r,l}$, i.e. $e_j \in \cap_{k=r}^{l} \mathcal{C}_{jk},\ \forall t \geq t_0$.

Assume that $h_{jk}(e_j)$ are CBFs for (13). By [28], ensuring the forward invariance of $\mathcal{C}_{jk}(e_j)$ yields that u_j should respect to the following inequalities:

$$\begin{cases} \underbrace{(\delta_{jr} + \lceil e_{1j,0} \rfloor^0)\dot{\rho}_j - e_{2j}}_{L_f h_{jr}} + \underbrace{\alpha_{jr}((\delta_{jr} + \lceil e_{1j,0} \rfloor^0)\rho_j - \rho_{j,\infty}\lceil e_{1j,0} \rfloor^0 - e_{1j})}_{\alpha_{jr}(h_{jr})} \geq 0, \\ \underbrace{(\delta_{jl} - \lceil e_{1j,0} \rfloor^0)\dot{\rho}_j + e_{2j}}_{L_f h_{jl}} + \underbrace{\alpha_{jl}((\delta_{jl} - \lceil e_{1j,0} \rfloor^0)\rho_j + \rho_{j,\infty}\lceil e_{1j,0} \rfloor^0 + e_{1j})}_{\alpha_{jl}(h_{jl})} \geq 0, \end{cases} \tag{16}$$

where $\alpha_{jr}(\cdot)$ and $\alpha_{jl}(\cdot)$ are class $\mathcal{K}$ functions of their arguments. It can be noticed that $L_g h_{jr} = 0$ and $L_g h_{jl} = 0$, and Eq. (16) are invalid constraints for u_j with respect to (13). Obviously, these CBFs are not capable of synthesizing controllers and guarantee the forward invariance of set $\mathcal{C}_{jk}$.

By introducing the definition of the relative degree in Def. 1, inequalities (4) have the relative degree of 2 with respect to (13). Thus, we select the HOCBFs of degree 2 to solve performance-critical control input sets for constraints (4). Before that, $\chi_{0jk}(e_j)$, $\chi_{1jk}(e_j)$, and $\chi_{2jk}(e_j)$ with $h_{jk}(e_j)$ are defined as

$$\begin{cases} \chi_{0jk}(e_j) = h_{jk}(e_j), \\ \chi_{1jk}(e_j) = (\frac{d}{dt} + \alpha_{1jk})\chi_{0jk}(e_j), \\ \chi_{2jk}(e_j) = (\frac{d}{dt} + \alpha_{2jk})\chi_{1jk}(e_j), \end{cases} \tag{17}$$

where $\alpha_{1jk}(\cdot)$ and $\alpha_{2jk}(\cdot)$ are $\mathcal{K}$ functions. From [29], the Lipschitz continuous controllers for ensuring the forward invariance of sets (14) should satisfy

$$\begin{aligned} & L_f^2 h_{jk}(e_j) + L_g L_f h_{jk}(e_j)(u_j + w_j) + \\ & \frac{\partial^2 h_{jk}(e_j)}{\partial t^2} + O(h_{jk}(e_j)) + \alpha_{2jk}(\chi_{1jk}(e_j)) \geq 0. \end{aligned} \tag{18}$$

where $O(\cdot)$ denotes the remaining Lie derivative along f and partial derivatives with respect to t with degree less than or equal to 1. However, the inequality constraints (18) on control input u_j cannot be solved directly due to the unknown term w_j. Therefore, it is essential to determine feasible constrained conditions that ensure the forward invariance of sets in (14) under unknown disturbance w_j.

Inspired by the ISSf in [30], an enlarged set $\mathcal{C}_{jk}^d$ satisfying $\mathcal{C}_{jk}^d \supset \mathcal{C}_{jk}$ is constructed as follows

$$\begin{aligned} \mathcal{C}_{jk}^d &= \{e_j \in \mathbb{R}^2 \mid h_{jk}(e_j) + \gamma_{jk}(\|w_j\|_\infty) \geq 0\}, \\ \partial\mathcal{C}_{jk}^d &= \{e_j \in \mathbb{R}^2 \mid h_{jk}(e_j) + \gamma_{jk}(\|w_j\|_\infty) = 0\}, \\ \mathrm{Int}(\mathcal{C}_{jk}^d) &= \{e_j \in \mathbb{R}^2 \mid h_{jk}(e_j) + \gamma_{jk}(\|w_j\|_\infty) > 0\}, \end{aligned}$$

where $\gamma_{jk}(\cdot) \in \mathcal{K}_\infty$. According to the definition of ISSf, the set $\mathcal{C}_{jk}$ is ISSf if the set $\mathcal{C}_{jk}^d$ is forward invariant.

For $h_{jk}(e_j)$ of relative degree 2, we define the sets $\mathcal{C}^d_{1jk}$ and $\mathcal{C}^d_{2jk}$ associated with (17) for the system (13)

$$\mathcal{C}^d_{1jk} = \{e_j \in \mathbb{R}^2 \mid \Phi_{1jk} \geq 0\},\ \mathcal{C}^d_{2jk} = \{e_j \in \mathbb{R}^2 \mid \Phi_{2jk} \geq 0\} \tag{19}$$

with

$$\Phi_{1jk} = \chi_{0jk}(e_j) + \gamma_{1jk}(\|w_j\|_\infty),\ \Phi_{2jk} = \chi_{1jk}(e_j) + \gamma_{2jk}(\|w_j\|_\infty), \tag{20}$$

where $\gamma_{1jk}(\cdot)$ and $\gamma_{2jk}(\cdot)$ are $\mathcal{K}_\infty$ functions. The following lemma states that set $\mathcal{C}^d_{1jk} \cap \mathcal{C}^d_{2jk}$ is forward invariant, and the set $\mathcal{C}_{jk}$ is ISSf.

Lemma 3: Let functions $\chi_{0jk}(e_j)$, $\chi_{1jk}(e_j)$, $\chi_{2jk}(e_j)$ be defined by (17) and sets $\mathcal{C}^d_{1jk}$, $\mathcal{C}^d_{2jk}$ be defined by (19). A continuous differentiable function h_{jk} of degree 2 for (13) is called as an input-to-state safe high order barrier function (ISSf-HOBF) if there exist a constant $\iota \in \mathbb{R}^+$, a class $\mathcal{K}_\infty$ function $\iota_{jk}(\cdot)$, and differentiable class $\mathcal{K}_{\infty,e}$ functions $\gamma_{1jk}(\cdot)$ and $\gamma_{2jk}(\cdot)$ such that

$$\chi_{2jk}(e_j) \geq -\iota_{jk}(\|w_j\|_\infty), \forall t \geq t_0. \tag{21}$$

for $e_j \in \mathbb{R}^m$ and $w_j \in \mathbb{R}^m$ with $\|w_j\|_\infty \leq \iota$. The ISSf-HOBF h_{jk} renders that the set $\mathcal{C}_{jk}$ is ISSf.

Proof: If $h_{jk}(e_j)$ is ISSf-HOBF, the derivatives of Φ_{2jk} from (20)-(21) is taken as

$$\dot{\Phi}_{2jk} \geq -\alpha_{2jk}(\Phi_{2jk} - \gamma_{2jk}(\|w_j\|_\infty)) - \iota_{jk}(\|w_j\|_\infty). \tag{22}$$

Similar to the proof of [28, Proposition 1], consider a set $\partial\mathcal{C}^d_{2jk}$, which yields $\Phi_{2jk} = 0$ for $e_j \in \partial\mathcal{C}^d_{2jk}$.

Then, one has $\dot{\Phi}_{2jk} \geq -\alpha_{2jk}(-\gamma_{2jk}(\|w_j\|_\infty)) - \iota_{2jk}(\|w_j\|_\infty)$. By choosing $\gamma_{2jk}(\cdot) = -\alpha^{-1}_{2jk}(\cdot) \circ \iota_{2jk}(\cdot)$, it follows that $\dot{\Phi}_{2jk} \geq 0$ for $\Phi_{2jk} = 0$. By Lemma 1, it gets that the set $\mathcal{C}^d_{2jk}$ is forward invariant, i.e.

$$\chi_{1jk}(e_{1j}) \geq -\gamma_{2jk}(\|w_j\|_\infty),\ \forall t \geq t_0. \tag{23}$$

From (20) and (23), it yields that the set $\mathcal{C}^d_{1jk}$ is forward invariant, i.e.

$$\chi_{0jk}(e_{1j}) \geq -\gamma_{1jk}(\|w_j\|_\infty),\ \forall t \geq t_0. \tag{24}$$

Hence, it is concluded that $\mathcal{C}^d_{1jk} \cap \mathcal{C}^d_{2jk}$ is forward invariant. According to [30, Definition 6], the system (13) is ISSf on $\mathcal{C}_{jk}$ defined by (14).

Lemma 4: Given a set $\mathcal{C}_{jk}$ in (14) with a continuously differentiable function h_{jk} of degree 2 for system (13), h_{jk} is an input-to-state safe high order control barrier function (ISSf-HOCBF) if there exist constants $\varepsilon_{jk}, \iota \in \mathbb{R}^+$ and a function $\alpha_{2jk}(\cdot) \in \mathcal{K}_\infty$ such that

$$\sup_{u_j \in \mathbb{R}} \left\{ L_f\chi_{1jk} + L_g\chi_{1jk}u_j - \varepsilon^2_{jk}\frac{\partial\chi_{1jk}}{\partial e_j^T} g_j g_j^T \frac{\partial\chi_{1jk}}{\partial e_j} + \alpha_{2jk}(\chi_{1jk}) \geq 0 \right\}, \tag{25}$$

for $\forall e_j \in \mathbb{R}^2$ and $w_j \in \mathbb{R}$ with $\|w_j\|_\infty \leq \iota$. With ISSf-HOCBF h_{jk}, a performance-critical control input set is presented as

$$\mathcal{U}_j = \left\{ u_j \mid L_f \chi_{1jk} + L_g \chi_{1jk} u_j - \varepsilon_{jk}^2 \frac{\partial \chi_{1jk}}{\partial e_j^T} g_j g_j^T \frac{\partial \chi_{1jk}}{\partial e_j} \geq -\alpha_{2jk}(\chi_{1jk}) \right\}. \quad (26)$$

And any continuous controller $u_j \in \mathcal{U}_j$ for $\forall e_j \in \mathbb{R}^m$ ensures that $\mathcal{C}_{jk}$ is ISSf.
Proof: Taking the derivative of χ_{1jk} along (13) yields that

$$\dot{\chi}_{1jk} = L_{f_j} \chi_{1jk} + L_{g_j} \chi_{1jk} u_j + \frac{\partial \chi_{1jk}}{\partial e_j^T} g_j w_j. \quad (27)$$

For $u_j \in \mathcal{U}_j$, it follows that

$$\dot{\chi}_{1jk} \geq \varepsilon_{jk}^2 \frac{\partial \chi_{1jk}}{\partial e_j^T} g_j g_j^T \frac{\partial \chi_{1jk}}{\partial e_j} + \frac{\partial \chi_{1jk}}{\partial e_j^T} g_j w_j - \alpha_{2jk}(\chi_{1jk}). \quad (28)$$

Using (17), one has

$$\chi_{2jk} \geq \|\varepsilon_{jk} \frac{\partial \chi_{1jk}}{\partial e_j^T} g_j + \frac{1}{2\varepsilon_{jk}} w_j\|^2 - \frac{1}{4\varepsilon_{jk}^2} \|w_j\|^2 \geq -\frac{1}{4\varepsilon_{jk}^2} \|w_j\|^2. \quad (29)$$

From Lemma 3, it means that function h_{jk} is an ISSf-HOCBF of degree 2 for system (13). Then, the controller $u_j \in \mathcal{U}_j$ can guarantee that set $\mathcal{C}_{jk}$ is ISSf, i.e. PPC objective (4) is satisfied.

The proposed nominal controller (11) does not consider the performance requirements. To obtain the user-specified performance, the control input constraint in (26) needs to be prior satisfied over the nominal controller (11), which is referred to as "performance-critical". Then, a quadratic program is formulated to solve the modified controller $u_{\text{opt},j}$

$$\begin{aligned} u_{j\text{opt}} = \arg \min_{u_j \in \mathbb{R}} \ & \|u_j - u_{j\text{nom}}\|^2 \\ \text{s.t. } & A_j u_j \leq B_j \end{aligned} \quad (30)$$

with $A_j = -L_{g_j} \chi_{1jk},\ B_j = L_{f_j} \chi_{1jk} - \varepsilon_{jk}^2 \frac{\partial \chi_{1jk}}{\partial e_j^T} g_j g_j^T \frac{\partial \chi_{1jk}}{\partial e_j} + \alpha_{2jk}(\chi_{1jk})$.

4 SIMULATION EXAMPLE

In this section, a simulation example of an unmanned surface vehicle is provided to evaluate the effectiveness of the proposed observer-based performance-critical control method. As shown in Fig. 1, the motion dynamics of the unmanned surface vehicle is described as follows

$$\begin{cases} \dot{\eta} = \vartheta, \\ \dot{\vartheta} = \dot{R}(\psi) R^T(\psi) \vartheta - C' \vartheta - D' \vartheta + R(\psi) M^{-1} \tau, \\ C' = R(\psi) M^{-1} C R^T(\psi), \\ D' = R(\psi) M^{-1} D R^T(\psi), \\ R(\psi) = \left[\cos\psi \ -\sin\psi \ 0; \ \ \sin\psi \ \cos\psi \ 0; \ \ 0 \ 0 \ 1 \right] \end{cases} \quad (31)$$

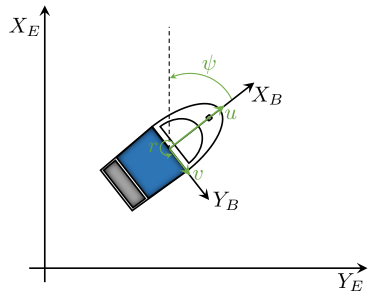


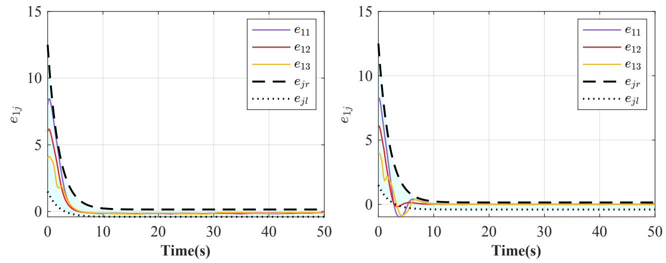
Fig. 1. Reference frames: $X_E - Y_E$ and $X_B - Y_B$.

where $\eta \in \mathbb{R}^3$ denotes a position and heading vector in reference frame $X_E - Y_E$; $\vartheta = R(\psi)\nu$ with $\nu \in \mathbb{R}^3$ being a surge, sway, and yaw velocity vector in $X_B - Y_B$; $M \in \mathbb{R}^{3\times3}$, $C \in \mathbb{R}^{3\times3}$, and $D \in \mathbb{R}^{3\times3}$ are the vehicle parameter matrices satisfying $M = M^T$ and $C = -C^T$, refer to [31] for details. Simulation parameters are select as $u_{jd} = \mathrm{col}(\sin(t), 0.4\cos(0.5t), 0)$, $\rho_{j0} = 10$, $\rho_{j\infty} = 0.5$, $\delta_{jr} = 0.3$, $\delta_{jl} = 0.8$, $\sigma_j = 0.5$, $L_1^o = \mathrm{diag}\{2.266, 2.266, 2.266\}$, $L_2^o = diag\{3.85, 3.85, 3.85\}$, $L_1^z = \mathrm{diag}\{2, 2, 2\}$, $L_2^z = \mathrm{diag}\{3, 3, 3\}$, $L^c = \mathrm{diag}\{1, 1, 1\}$, $\alpha_{1jk} = \alpha_{2jk} = 1$, $\varepsilon_{jk} = 0.5$. To illustrate the effectiveness of proposed method, we conduct a comparison simulation by using the nominal controller (11) and the modified controller (30), respectively.

Comparison results are depicted in Figs. 2-5. Specifically, Fig. 2 displays tracking errors e_{1j}, e_{2j} with the modified controller (30) and the nominal controller (11), respectively. From Figs. 2(a)-2(b), it is observed that e_{1j} under modified controller $u_{j\mathrm{opt}}$ can evolve within given performance bounds e_{jr} and e_{jl}, whereas e_{1j} under $u_{j\mathrm{nom}}$ exhibits obvious overshoot and violates the constraints e_{jl}. Further, Fig. 3 shows the error e_{2j} under $u_{j\mathrm{opt}}$ and $u_{j\mathrm{nom}}$, and it is seen that the transient performance of e_{2j} under $u_{j\mathrm{opt}}$ is superior to that under $u_{j\mathrm{nom}}$. According to Fig. 4, it gets that sets $\mathcal{C}_{jk}$ defined by $h_{jk}(e_j)$ for system (13) with unknown disturbances are ISSf by using the controller satisfying (26), i.e. $u_j \in \mathcal{U}_j$. In addition, it is concluded that the constraints (4) are not violated. And Fig. 3 plots the nominal and modified control inputs.

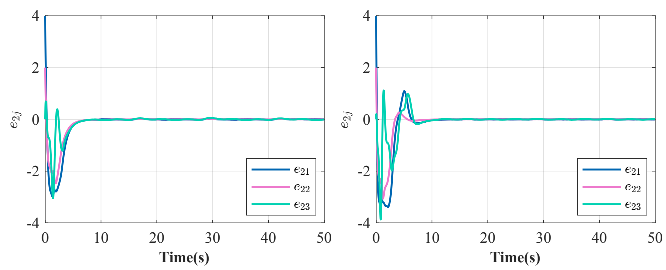
5 CONCLUSIONS

In this paper, we developed an observer-based performance-critical control method for uncertain MIMO nonlinear systems in Brunovsky form. By using the designed FTESO, the unknown term can be estimated within a finite time. With estimated values from FTESO, a nominal controller was presented by using the integral sliding surface technique. Next, we constructed a 2nd-order nonlinear system with unknown input disturbances. ISSf-HOCBfs of relative degree 2 associated with performance constraints were designed to acquire the ISSf control input sets under unknown disturbances. By using the nominal controller and ISSf



(a) With modified controllers. (b) With nominal controllers.

Fig. 2. Tracking errors e_{1j} under modified and nominal controllers.



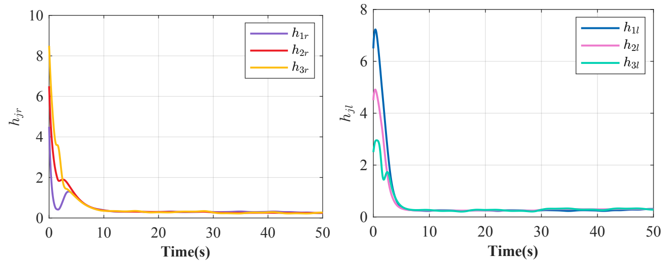
(a) With modified controllers. (b) With nominal controllers.

Fig. 3. Tracking errors e_{2j} under modified and nominal controllers.

control input sets, tracking errors were ensured to satisfy the user-prescribed constraints. The effectiveness of the proposed method was demonstrated through a simulation example of an unmanned surface vehicle.

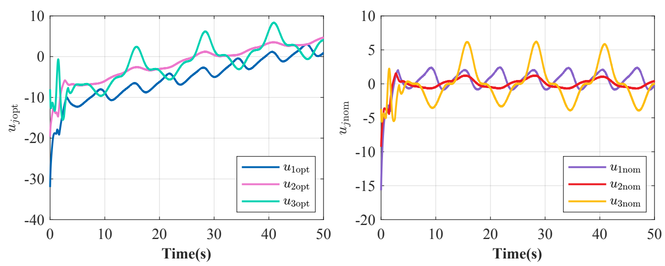
References

1. Z. Peng, J. Wang, D. Wang, and Q. Han, "An overview of recent advances in coordinated control of multiple autonomous surface vehicles," *IEEE Trans. Ind. Informat.*, vol. 17, no. 2, pp. 732–745, Feb. 2021.
2. S. Shao and M. Chen, "Adaptive neural discrete-time fractional-order control for a UAV system with prescribed performance using disturbance observer," *IEEE Trans. Syst. Man, Cybern., Syst.*, vol. 51, no. 2, pp. 742–754, 2018.
3. Z. Peng, D. Wang, and J. Wang, "Data-driven adaptive disturbance observers for model-free trajectory tracking control of maritime autonomous surface ships," *IEEE Trans. Neural Netw. Learn. Syst.s*, vol. 32, no. 12, pp. 5584–5594, 2021.
4. L. Qiao and W. Zhang, "Double-loop integral terminal sliding mode tracking control for UUVs with adaptive dynamic compensation of uncertainties and disturbances," *IEEE J. Ocean. Eng.*, vol. 44, no. 1, pp. 29–53, 2019.
5. S.-L. Dai, K. Lu, and X. Jin, "Fixed-time formation control of unicycle-type mobile robots with visibility and performance constraints," *IEEE Trans. Ind. Electron.*, vol. 68, no. 12, pp. 12 615–12 625, 2020.



(a) Based on upper bounds. (b) Based on lower bounds.

Fig. 4. The ISSf-HOCBFs based on performance constraints.



(a) Modified inputs. (b) Nominal inputs.

Fig. 5. The modified and nominal control inputs.

6. Y. Ouyang, L. Dong, and C. Sun, "Critic learning-based control for robotic manipulators with prescribed constraints," *IEEE Trans. Cybern.*, vol. 52, no. 4, pp. 2274–2283, 2020.
7. C. P. Bechlioulis and G. A. Rovithakis, "Robust adaptive control of feedback linearizable mimo nonlinear systems with prescribed performance," *IEEE Trans. Automat. Contr.*, vol. 53, no. 9, pp. 2090–2099, 2008.
8. S. He, C. Dong, S.-L. Dai, and T. Zou, "Cooperative deterministic learning and formation control for underactuated usvs with prescribed performance," *Int. J. Robust Nonlinear Control.*, vol. 32, no. 5, pp. 2902–2924, 2022.
9. W. Wu, R. Ji, W. Zhang, and Y. Zhang, "Transient-reinforced tunnel coordinated control of underactuated marine surface vehicles with actuator faults," *IEEE trans. Intell. Transp. Syst.*, 2023, Accepted.
10. R. Ji, B. Yang, J. Ma, and S. S. Ge, "Saturation-tolerant prescribed control for a class of mimo nonlinear systems," *IEEE Trans. Cybern.*, 2021.
11. W. Wu, Y. Zhang, W. Zhang, and W. Xie, "Distributed finite-time performance-prescribed time-varying formation control of autonomous surface vehicles with saturated inputs," *Ocean Eng.*, vol. 266, p. 112866, Oct. 2022.
12. M. Chen, "Disturbance attenuation tracking control for wheeled mobile robots with skidding and slipping," *IEEE Trans. Ind. Electron.*, vol. 64, no. 4, pp. 3359–3368, 2016.
13. Z. Jia, Z. Hu, and W. Zhang, "Adaptive output-feedback control with prescribed performance for trajectory tracking of underactuated surface vessels," *ISA Trans.*, vol. 95, pp. 18–26, 2019.

14. Y. Zhang, S.-H. Wang, B. Chang, and W.-H. Wu, “Adaptive constrained backstepping controller with prescribed performance methodology for carrier-based UAV,” *Aerosp. Sci. Technol.*, vol. 92, pp. 55–65, 2019.
15. H. Liang, Y. Zhang, T. Huang, and H. Ma, “Prescribed performance cooperative control for multiagent systems with input quantization,” *IEEE Trans. Cybern.*, vol. 50, no. 5, pp. 1810–1819, 2019.
16. B. S. Park and S. J. Yoo, “Robust trajectory tracking with adjustable performance of underactuated surface vessels via quantized state feedback,” *Ocean Eng.*, vol. 246, p. 110475, 2022.
17. A. D. Ames, J. W. Grizzle, and P. Tabuada, “Control barrier function based quadratic programs with application to adaptive cruise control,” in *Proc. of 53rd IEEE Conference on Decision and Control*. IEEE, 2014, pp. 6271–6278.
18. X. Xu, J. W. Grizzle, P. Tabuada, and A. D. Ames, “Correctness guarantees for the composition of lane keeping and adaptive cruise control,” *IEEE Trans. Autom. Sci. Eng.*, vol. 15, no. 3, pp. 1216–1229, 2017.
19. L. Wang, A. D. Ames, and M. Egerstedt, “Safety barrier certificates for collisions-free multirobot systems,” *IEEE Trans. Robot.*, vol. 33, no. 3, pp. 661–674, 2017.
20. W. Wu, Y. Zhang, W. Zhang, and W. Xie, “Output-feedback finite-time safety-critical coordinated control of path-guided marine surface vessels based on neurodynamic optimization,” *IEEE Trans. Syst. Man, Cybern., Syst.*, vol. 53, no. 3, pp. 1788–1800, 2023.
21. Z. Peng, C. Wang, Y. Yin, and J. Wang, “Safety-certified constrained control of maritime autonomous surface ships for automatic berthing,” *IEEE Trans. Veh. Technol.*, 2023, doi:10.1109/TVT.2023.3253204.6.
22. W. Wu, D. Wu, Y. Zhang, S. Chen, and W. Zhang, “Safety-critical trajectory tracking for mobile robots with guaranteed performance,” *IEEE/CAA J. Autom. Sinica*, 2023.
23. S. Kolathaya and A. D. Ames, “Input-to-state safety with control barrier functions,” *IEEE Contr. Syst. Lett.*, vol. 3, no. 1, pp. 108–113, 2019.
24. S. Gao, Z. Peng, H. Wang, L. Liu, and D. Wang, “Safety-critical model-free control for multi-target tracking of USVs with collision avoidance,” *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 7, pp. 1323–1326, 2022.
25. H. K. Khalil, “Nonlinear systems third edition,” *Patience Hall*, vol. 115, 2002.
26. C. Wang and D. J. Hill, “Learning from neural control,” *IEEE Transactions on Neural Networks*, vol. 17, no. 1, pp. 130–146, 2006.
27. A. Dávila, J. A. Moreno, and L. Fridman, “Optimal lyapunov function selection for reaching time estimation of super twisting algorithm,” in *Proc. of 48th IEEE Conference on Decision and Control*. IEEE, 2009, pp. 8405–8410.
28. A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, “Control barrier function based quadratic programs for safety critical systems,” *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 3861–3876, 2016.
29. W. Xiao and C. Belta, “Control barrier functions for systems with high relative degree,” in *Proc. of 58th IEEE Conference on Decision and Control*. IEEE, 2019, pp. 474–479.
30. A. Alan, A. J. Taylor, C. R. He, G. Orosz, and A. D. Ames, “Safe controller synthesis with tunable input-to-state safe control barrier functions,” *IEEE Contr. Syst. Lett.*, vol. 6, pp. 908–913, 2021.
31. T. I. Fossen, *Handbook of marine craft hydrodynamics and motion control*. John Wiley & Sons, 2011.